$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right) \quad ; \quad a_0 = 15$$

$$2 a_{n+1} a_n = a_n^2 + 9$$

$$a_n^2 - 2 a_n . a_{n+1} + 9 = 0$$

it is quadratic equation and a_n is seqⁿ of real number so

$$D = b^2 - 4ac \geq 0$$

$$4 a_{n+1}^2 - 4.9 \geq 0$$

$$a_{n+1}^2 \geq 9$$

$$\Rightarrow \quad a_{n+1} \geq 3$$

as $\quad a_2 > 3$

$$a_n > 0 \quad , \forall n \in \mathbb{N}$$

now $\quad a_{n+1} - a_n = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right) - a_n$

$$= \frac{a_n}{2} + \frac{9}{2a_n} - a_n$$

$$= \frac{9}{2a_n} - \frac{a_n}{2}$$

$$= \frac{18 - a_n^2}{2a_n} \leq 0$$

$$a_{n+1} - a_n \leq 0$$

$$\Rightarrow \quad a_{n+1} \leq a_n$$

$\Rightarrow$ monotonic seqⁿ

& $a_n \geq 3 \quad \Rightarrow$

sequence bounded + monotonic $\Rightarrow$ seqn is convergent

So let $\lim_{n \to \infty} a_n = l \Rightarrow \lim_{n \to \infty} a_{n+1} = l$

So

$$l = \frac{1}{2}\left(l + \frac{9}{l}\right)$$

$$2l^2 = l^2 + 9$$

$$l^2 = 9$$

$$l = \pm 3$$

$$\underline{\underline{l = 3}}$$