$$a_{n+1} = 2 - \frac{1}{2+a_n} \quad ; \; a_0 = 5$$

$n = 0$

$$a_1 = 2 - \frac{1}{2+a_0} = 2 - \frac{1}{2+5} = 2 - \frac{1}{7}$$

$$a_1 = \frac{13}{7} < 2$$

$n = 1$; $a_2 = 2 - \frac{1}{2+a_1} = 2 - \frac{1}{2+\frac{13}{7}} = 2 - \frac{7}{27} = \frac{47}{27} < 2$

$n = 2$ $\quad a_3 = 2 - \frac{1}{2+a_2} = 2 - \frac{1}{2+\frac{47}{27}} = 2 - \frac{27}{101} = \frac{202-27}{101}$

$$a_3 = \frac{175}{101} < 2$$

|

|

similarly you get

$a_0 > a_1 > a_2 > a_3 > a_5 > a_5 - - -$

$\langle a_n \rangle$ is monotonic decreasing

$$\underbrace{0 < a_n < 2}$$

Sequence is bounded

monotonic + bounded = convergent

Let $\lim_{n \to \infty} a_n = l$

so $\lim_{n \to \infty} a_{n+1} = l$

so $\lim_{n \to \infty} a_{n+1} = \lim_{n \to \infty} \left(2 - \frac{1}{2 + a_n}\right)$

$$l = 2 - \frac{1}{2 + l}$$

$$(2+l)l = 2(2+l) - 1$$

$$\Rightarrow 2l + l^2 = 4 + 2l - 1$$

$$l^2 = 3$$

$$l = \pm\sqrt{3}$$

$$\underline{\underline{l = \sqrt{3}}}$$

$l = -\sqrt{3}$ is not possible
bcoz sequence
is positive