

$$a_n = \begin{cases} n^3 2^{-n} & ; n \text{ is odd} \\ \frac{1}{3^n} & ; n \text{ is even} \end{cases}$$

① when n is odd

$$\sum a_n = \sum \frac{n^3}{2^n}$$

then

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{(n+1)^3}{2^{n+1}} \times \frac{2^n}{n^3}$$

$$= \lim_{n \rightarrow \infty} \frac{n^3 \left(1 + \frac{1}{n}\right)^3}{2^{n+1} \cdot 2^{-n}} \times \frac{2^n}{n^3}$$

$$= \frac{1}{2} < 1$$

hence by d'Alembert's ratio test
series is convergent

⑦ When n is odd even

$$\text{then } \sum a_n = \sum \frac{1}{3^n}$$

By ratio test

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{1}{3^{n+1}} \times 3^n$$

$$= \frac{1}{3} < 1$$

hence series is convergent

and sum of two convergent series is again a convergent series