

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \dots$$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots$$

$$\log(1+x) - \log(1-x) = 2 \left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \right]$$

Put $x = \frac{1}{2}$ on both sides we get

$$\log\left(\frac{3}{2}\right) - \log\left(\frac{1}{2}\right) = 2 \left(\frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2^3} + \frac{1}{5} \cdot \frac{1}{2^5} + \dots \right)$$

$$\log(3) = 1 + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{4^2} + \frac{1}{7} \cdot \frac{1}{4^3} + \dots$$