

$$f(x, y) = \begin{cases} x^2 + y^2 & x, y \in \mathbb{R}^c \\ 0 & \text{otherwise} \end{cases}$$

Continuity at (0,0)

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$$

clearly continuous at (0,0)

[Also we can verify by taking path passing through (0,0)]

Differentiability at (0,0)

$$f_x(0,0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2}{h} = 0$$

$$f_y(0,0) = 0$$

$$\Rightarrow \frac{f(h,k) - f(0,0) - f_x(0,0) \cdot h - f_y(0,0) \cdot k}{\sqrt{h^2 + k^2}} = g(h,k) \rightarrow \text{should be exist}$$

$$\Rightarrow \frac{h^2 + k^2 - 0 - 0}{\sqrt{h^2 + k^2}} = g(h,k) = \sqrt{h^2 + k^2}$$

$$\lim_{(x,y) \rightarrow (0,0)} g(h,k) = 0 \rightarrow \text{exist}$$

Differentiable at (0,0)

$\Rightarrow$ But f is continuous only at $(0,0)$

$\Rightarrow$ f is not differentiable on $\mathbb{R} \setminus (0,0)$

So option (c) is correct