

$$2 \frac{dy}{dx} + 3y = \sin 2x$$

$$; y(1)=6, y(2)=7$$

$$\Rightarrow \frac{dy}{dx} + \frac{3}{2}y = \frac{\sin 2x}{2}$$

it is a linear differential equation hence it's integrating factor

$$I.o.F = e^{\int \frac{3}{2} dx} = e^{\frac{3}{2}x}$$

$$y \cdot (e^{\frac{3}{2}x}) = \int \frac{\sin 2x}{2} e^{\frac{3}{2}x} dx + C$$

$$\because \sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \text{So} \quad \sin 2x = \frac{e^{i2x} - e^{-i2x}}{2i}$$

$$y e^{\frac{3}{2}x} = \frac{1}{2} \int \frac{e^{i2x} - e^{-i2x}}{2i} e^{\frac{3}{2}x} dx + C$$

$$= \frac{1}{4i} \left[\int (e^{(\frac{3}{2} + 2i)x} - e^{(\frac{3}{2} - 2i)x}) dx \right] + C$$

$$y e^{\frac{3}{2}x} = \frac{1}{4i} \left[\frac{e^{(\frac{3}{2} + 2i)x}}{(\frac{3}{2} + 2i)} - \frac{e^{(\frac{3}{2} - 2i)x}}{(\frac{3}{2} - 2i)} \right] + C$$

Now

$$y(x) = \frac{1}{4i} \left[\frac{e^{\frac{3}{2}x} e^{2ix}}{(\frac{3}{2} + 2i)} - \frac{e^{\frac{3}{2}x} e^{-2ix}}{(\frac{3}{2} - 2i)} \right] + C e^{-\frac{3}{2}x}$$

Now

$$y(0) = \frac{1}{4i} \left[\frac{e^0}{(\frac{3}{2}+2i)} - \frac{e^0}{(\frac{3}{2}-2i)} \right] + ce^0$$

$$6 = \frac{1}{4i} \left[\frac{\cancel{\frac{3}{2}}-2i - \cancel{\frac{3}{2}}-2i}{(\frac{9}{4}+4)} \right] + c$$

$$6 = \frac{1}{4i} \left[\frac{-4i}{(\frac{25}{4})} \right] + c$$

$$6 = -\frac{4}{25} + c \quad \Rightarrow \quad c = 6 + \frac{4}{25} = \frac{154}{25}$$

$$c = \frac{154}{25}$$

Hence

$$y(x) = \frac{1}{4i} \left[\frac{e^{2ix}}{\frac{3}{2}+2i} - \frac{e^{-2ix}}{\frac{3}{2}-2i} \right] + \frac{154}{25} e^{-3/2x}$$

$$= \frac{1}{4i} \left[\frac{(\frac{3}{2}-2i) e^{2ix} - e^{-2ix}(\frac{3}{2}+2i)}{(\frac{9}{4}+4)} \right] + \frac{154}{25} e^{-3/2x}$$

$$= \frac{1}{25} \times \frac{1}{4i} \left[\frac{3}{2} (e^{2ix} - e^{-2ix}) - 2i (e^{2ix} + e^{-2ix}) \right] + \frac{154}{25} e^{-3/2x}$$

$$y(x) = \frac{1}{25} \left[3 \left(\frac{e^{2ix} - e^{-2ix}}{2i} \right) - 4 \left(\frac{e^{2ix} + e^{-2ix}}{2} \right) \right] + \frac{154}{25} e^{-3/2x}$$

$$y(x) = \frac{1}{25} [3 \sin 2x - 4 \cos 2x] + \frac{154}{25} e^{-3/2x}$$

$$y(2) = \frac{1}{25} [3 \sin 4 - 4 \cos 4] + \frac{154}{25} e^{-3}$$

$$= 0.1554$$

$$y(2) = \underline{0.16} \quad \text{Ans}$$