Expansion of $\tan^{-1}(x)$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots$$

$$\tan^{-1}\left(\frac{1}{n^2+n+1}\right) = \frac{1}{(n^2+n+1)} - \frac{1}{(n^2+n+1)^3 \cdot 3} + \frac{1}{(n^2+n+1)^5 \cdot 5} - \cdots$$

$$\sum \tan^{-1}\left(\frac{1}{n^2+n+1}\right) = \sum \frac{1}{n^2}\left[\frac{1}{(1+\frac{1}{n}+\frac{1}{n^2})} - \frac{1}{n^4(1+\frac{1}{n}+\frac{1}{n^2})^3 \cdot 3} + \cdots\right)$$

Let an auxiliary equation $\sum v_n = \sum \frac{1}{n^2}$ such that

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{\frac{1}{n^2}\left[\frac{1}{(1+\frac{1}{n}+\frac{1}{n^2})} - \frac{1}{3n^4(1+\frac{1}{n}+\frac{1}{n^2})^3} + \cdots\right)}{\frac{1}{n^2}}$$

$$= 1 \neq 0$$

Now $\sum v_n = \sum \frac{1}{n^2}$; $p = 2 > 1$ is convergent

hence $\sum \tan^{-1}\left(\frac{1}{n^2+n+1}\right)$ is also convergent