

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$$

$$\det(A) = 3 \times 5 - 4 \times 1 = 15 - 4 = 11$$

$$\det(A) = 11 \equiv 4 \pmod{7}$$

and

$$\text{adj}(A) = \begin{bmatrix} 5 & -1 \\ -4 & 3 \end{bmatrix}$$

-1 is the inverse of 1

$$-1 + 1 = 0$$

8 -4 is also the inverse of 4
 $-4 + 4 = 0$

So

$$\text{adj}(A) = \begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A) = \frac{1}{4} \begin{pmatrix} 5 & 1 \\ 4 & 3 \end{pmatrix}$$

$$= 4^{-1} \begin{pmatrix} 5 & 1 \\ 4 & 3 \end{pmatrix}$$

$\frac{1}{4}$ inverse of 4 $(\cancel{\frac{1}{4}} \times 4 = 1)$

$$4 \times ? = 1$$

$$4 \times 2 = 1 \pmod{7}$$

$$4^{-1} = 2$$

$$A^{-1} = 2 \begin{pmatrix} 5 & 1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 10 & 2 \\ 8 & 3 \end{pmatrix}$$