

$$\sum \frac{1}{2^n} \quad ; \quad q_n = \frac{n}{2^n} \quad ; \quad n \text{ is odd}$$

$$\frac{1}{2^n} \quad ; \quad n \text{ is even}$$

Case - I When n is even then

$$\sum q_n = \sum \frac{1}{2^n} \quad ; \quad q_n = \frac{1}{2^n}$$

$$\lim_{n \rightarrow \infty} \frac{q_{n+1}}{q_n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{2^{n+1}}}{\frac{1}{2^n}} = \lim_{n \rightarrow \infty} \frac{1}{2^{n+1}} \times 2^n = \frac{1}{2}$$

$$\lim_{n \rightarrow \infty} \frac{q_{n+1}}{q_n} = \frac{1}{2} < 1$$

$\Rightarrow \sum q_n = \sum \frac{1}{2^n}$ is convergent series when n is even

Case - II When n is odd

$$\sum q_n = \sum \frac{n}{2^n} \quad ; \quad \text{by ratio test}$$

$$\lim_{n \rightarrow \infty} \frac{q_{n+1}}{q_n} = \lim_{n \rightarrow \infty} \frac{(n+1)}{2^{n+1}} \times \frac{2^n}{n} = \lim_{n \rightarrow \infty} \frac{n(1+\frac{1}{n})}{2^{n+1}} \times \frac{2^n}{n}$$

$$= \frac{(1+0)}{2}$$

$$= \frac{1}{2} < 1$$

$\Rightarrow \sum q_n = \sum \frac{n}{2^n}$ is also convergent

$\therefore$ Sum of two convergent series is also a convergent series

hence $\sum q_n$ is also convergent

$$\sum q_n \quad ; \quad q_n = \frac{1}{\log n} \quad ; \quad n > 1$$

$$\therefore n > \log n \quad ; \quad \forall n > 2$$

$$\Rightarrow \frac{1}{n} < \frac{1}{\log n}$$