Q $\sum_{n=1}^{\infty} \frac{4^n (n!)^2}{(2n)!}$; $u_n = \frac{4^n (n!)^2}{(2n)!}$

The Necessary condition for any series to convergent is

$$\lim_{n \to \infty} u_n = 0$$

but here $\lim_{n \to \infty} u_n = \lim_{n \to \infty} \frac{4^n \, n! \cdot n!}{2n!} = \lim_{n \to \infty} \frac{4^n}{\frac{2n!}{n! \cdot n!}}$

$$= \lim_{n \to \infty} \frac{4^n}{\frac{2n!}{(2n-n)! \, n!}} \qquad {}^nC_r = \frac{n!}{(n-r)! \, r!}$$

$$\lim_{n \to \infty} u_n = \lim_{n \to \infty} \frac{4^n}{({}^{2n}C_n)}$$

bcoz the radius of convergence of 4^n is very high to $({}^{2n}C_n)$

$$= \infty$$

$$\neq 0$$

$\Rightarrow$ Σu_n is divergent series

so $\Sigma \frac{4^n \, n! \, n!}{2n}$ is divergent

2. $\sum a_n$; $a_n = \frac{n}{2^n}$; n is odd

$\frac{1}{2^n}$; n is even

Case-I when n is even then

$\sum a_n = \sum \frac{1}{2^n}$; $a_n = \frac{1}{2^n}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1/2^{n+1}}{1/2^n} = \lim_{n\to\infty} \frac{1}{2^n \cdot 2} \times 2^n = \frac{1}{2}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{2} < 1$$

$\Rightarrow$ $\sum a_n = \sum \frac{1}{2^n}$ is convergent series when n is even

Case-II when n is odd

$\sum a_n = \sum \frac{n}{2^n}$; by ratio test

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)}{2^{n+1}} \times \frac{2^n}{n} = \lim_{n\to\infty} \frac{\cancel{n}(1+1/n)}{\cancel{2^n}\cdot 2} \times \frac{\cancel{2^n}}{\cancel{n}}$$

$$= \frac{(1+0)}{2}$$

$$= \frac{1}{2} < 1$$

$\Rightarrow$ $\sum a_n = \sum \frac{n}{2^n}$ is also convergent

$\because$ sum of two convergent series is also a convergent series

hence $\sum a_n$ is also convergent

3. $\sum a_n$; $a_n = \frac{1}{\log n}$; $n \geq 1$

$\because$ $n > \log n$; $\forall n > 2$

$\Rightarrow$ $\frac{1}{n} < \frac{1}{\log n}$

$$\sum \frac{1}{n} < \sum \frac{1}{\log n}$$

by limit comparision test $\because \sum \frac{1}{n}$ is divergent $(p=1)$ hence

$\sum \frac{1}{\log n}$ is divergent

4 $\quad \sum a_n = \sum_{n=1}^{\infty} \frac{1}{2^n}$

which is convergent

hence

1 — divergent

2 — convergent

3 — Divergent

4 — convergent