Since given particular sol^n is $y = a + b\sin 2x \ln(\sec 2x + \tan 2x)$ — (1)

and we know that C.F. + P.I. is a sol^n of diff. eq^n.

$$\frac{dy}{dx} = 0 + 2b\cos 2x \ln(\sec 2x + \tan 2x) + b\sin 2x \frac{1}{(\sec 2x + \tan 2x)} (2\sec 2x \cdot \tan 2x + 2\sec^2 2x)$$

$$\frac{dy}{dx} = 2b\cos 2x \ln(\sec 2x + \tan 2x) + b\sin 2x \frac{(2\sec 2x + \tan 2x) \cdot 2\sec 2x}{(\sec 2x + \tan 2x)}$$

$$\frac{dy}{dx} = 2b\cos 2x \ln(\sec 2x + \tan 2x) + b\sin 2x \cdot 2\sec 2x$$

$$\frac{d^2y}{dx^2} = -2b\sin 2x \ln(\sec 2x + \tan 2x) + 2b\cos 2x \times 2\sec 2x + 2b\cos 2x \sec 2x + 2b\sin 2x \sec 2x \tan 2x$$

$$= -4b\sin 2x \ln(\sec 2x + 8b\cos 2x \sec 2x + 4b\sin 2x \sec 2x \tan 2x$$ — (2)

Now put values in eq^n

$$\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x$$

$$= 4\sec^2 2x$$

$$-4b\sin 2x \ln(\sec 2x + \tan 2x) + 8b\cos 2x \sec 2x + 4b\sin 2x \sec 2x \tan 2x + 4(a + b\sin 2x \ln(\sec 2x + \tan 2x)) = 4\sec^2 2x$$

$$4a + 8b\cos 2x \sec 2x + 4b\sin 2x \sec 2x \tan 2x = 4\sec^2 2x$$

$4a + 8b + 4b \sin 2x \sec 2x \tan 2x = 4 \sec^2 2x$

~~$4a+8b \neq 0$~~ $\quad 4a + 8b + 4b \sin 2x \cdot \sec 2x \cdot \frac{\sin 2x}{\cot 2x} = 4 \sec^2 2x$

~~$a+2b \neq 0$~~

$$4a + 8b + 4b \cdot \frac{\sin^2 2x}{\cos 2x} \cdot \sec 2x = 4 \sec^2 2x$$

$$4a + 8b + 4b \cdot \tan^2 2x = 4 \sec^2 2x$$

$$4a + 8b + 4b(\sec^2 2x - 1) = 4 \sec^2 2x$$

$$4b \sec^2 2x + 4a + 8b - 4b = 4 \sec^2 2x$$

$$4b \sec^2 2x + 4a + 4b = 4 \sec^2 2x$$

Compaire both side coefficient of $\sec^2 2x$

$4b = 1$

$b = \frac{1}{4}$

Constant term and $4a + 4b = 0$

$a + b = 0$