

Problem:

$$\int \frac{\ln\left(\frac{x-1}{x+1}\right)}{x^2-1} dx$$

Substitute $u = \ln\left(\frac{x-1}{x+1}\right) \longrightarrow$

$$\frac{du}{dx} = \frac{(x+1) \left(\frac{1}{x+1} - \frac{x-1}{(x+1)^2} \right)}{x-1} \quad (\text{steps}) \longrightarrow$$

$$dx = \frac{x-1}{(x+1) \left(\frac{1}{x+1} - \frac{x-1}{(x+1)^2} \right)} du:$$

$$\int \frac{ue^u}{\left(\left(-\frac{2}{e^u-1} - 1 \right)^2 - 1 \right) \left(\frac{\left(\frac{2}{e^u-1} + 2 \right) (e^u-1)^2}{4} - \frac{e^u-1}{2} \right)}$$

Simplify:

$$= \frac{1}{2} \int u \, du$$

Now solving:

$$\int u \, du$$

Apply power rule:

$$\begin{aligned} \int u^n \, du &= \frac{u^{n+1}}{n+1} \text{ with } n = 1: \\ &= \frac{u^2}{2} \end{aligned}$$

Plug in solved integrals:

$$\frac{1}{2} \int u \, du$$
$$= \frac{u^2}{4}$$

Undo substitution $u = \ln\left(\frac{x-1}{x+1}\right)$:

$$= \frac{\ln^2\left(\frac{x-1}{x+1}\right)}{4}$$

The problem is solved:

$$\int \frac{\ln\left(\frac{x-1}{x+1}\right)}{x^2 - 1} \, dx$$
$$= \frac{\ln^2\left(\frac{x-1}{x+1}\right)}{4} + C$$
