$$\int \frac{x\sin(x)+\cos(x)}{x\left(\cos(x)+x\right)}\,\mathrm{d}x$$

Rewrite:

$$=\int\left(\frac{\sin(x)-1}{\cos(x)+x}+\frac{1}{x}\right)\mathrm{d}x$$

Apply linearity:

$$=\int\frac{\sin(x)-1}{\cos(x)+x}\,\mathrm{d}x+\int\frac{1}{x}\,\mathrm{d}x$$

Now solving:

$$\int\frac{\sin(x)-1}{\cos(x)+x}\,\mathrm{d}x$$

Substitute $u=\cos(x)+x \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=1-\sin(x)$

(steps) $\longrightarrow \mathrm{d}x=\frac{1}{1-\sin(x)}\,\mathrm{d}u$:

$$=-\int\frac{1}{u}\,\mathrm{d}u$$

Now solving:

$$\int\frac{1}{u}\,\mathrm{d}u$$

This is a standard integral:

$$=\ln(u)$$

Plug in solved integrals:

$$-\int\frac{1}{u}\,\mathrm{d}u$$

$$=-\ln(u)$$

$$= -\ln(u)$$

Undo substitution $u = \cos(x) + x$:

$$= -\ln(\cos(x) + x)$$

Now solving:

$$\int \frac{1}{x} \, \mathrm{d}x$$

Use previous result:

$$= \ln(x)$$

Plug in solved integrals:

$$\int \frac{\sin(x) - 1}{\cos(x) + x} \, \mathrm{d}x + \int \frac{1}{x} \, \mathrm{d}x$$

$$= \ln(x) - \ln(\cos(x) + x)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{x \sin(x) + \cos(x)}{x\,(\cos(x) + x)} \, \mathrm{d}x$$

$$= \ln(|x|) - \ln(|\cos(x) + x|) + C$$