$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 0$; $y(0) = 1$, $\frac{dy}{dx}\left(\frac{\pi}{4}\right) = 0$; in the range $0 < x < \frac{\pi}{4}$

Ans

$$(m^2 + 2m + 17) = 0$$

$$m_2 \quad \frac{-2 \pm \sqrt{4 - 4(17)}}{2}$$

$$m = \frac{-2 \pm \sqrt{-64}}{2} = \frac{-2 \pm i8}{2}$$

$$m = -1 \pm 4i$$

~~C.F = $e^{-x}(C_1 \cos(\quad) + C_2$~~ $\quad C.F = [C_1 \cos 4x + C_2 \sin 4x]\, e^{-x}$

$$y(x) = e^{-x}(C_1 \cos 4x + C_2 \sin 4x)$$

Now $\quad y(0) = 1 \Rightarrow y(0) = e^0 [C_1 \cos 0 + C_2 \sin 0] \Rightarrow 1 = C_1$

So

$$y(x) = e^{-x}(\cos 4x + C_2 \sin 4x)$$

$$y'(x) = e^{-x}(-4\sin 4x + 4C_2 \cos 4x) - e^{-x}(\cos 4x + C_2 \sin 4x)$$

$$y'\left(\tfrac{\pi}{4}\right) = e^{-\pi/4}(-4\sin\pi + 4C_2 \cos\pi) - e^{-\pi/4}[\cos\pi + C_2 \sin\pi)$$

$$0 = e^{-\pi/4}(0 + C_2 4(-1)) - e^{-\pi/4}(-1 + C_2 \cdot 0$$

$$0 = e^{-\frac{\pi}{4}}(-4C_2 + 1)\, e^{\frac{\pi}{4}} \quad \because e^{-\pi/4} \neq 0 \; ; \; -4C_2 + 1 = 0$$

$$\boxed{C_2 = 1/4}$$

So

$$y(x) = e^{-x}\left(\cos 4x + \frac{1}{4}\sin 4x\right)$$