$$\lim_{n\to\infty} \frac{x^n}{n!}$$

Case I when $x = 1$ then $\lim_{n\to\infty} \frac{x^n}{n!} = \lim_{n\to\infty} \frac{1^n}{n!} = \lim_{n\to\infty} \frac{1}{n!} = 0$

Case - II when $x > 1$

$$\lim_{n\to\infty} \frac{x^n}{n!}$$

Radius of convergence of $x^n < n!$ then

$$\lim_{n\to\infty} \frac{x^n}{n!} = 0$$

because $\frac{x^n}{n!} < 1 \; ; \forall n \in N$

Case - III when $0 \le x < 1$ then

$$\lim_{n\to\infty} x^n \to 0$$

So $\lim_{n\to\infty} \frac{x^n}{n!} = 0$

Case - IV when ~~$0 < x < 1$~~ $x \le 0$

~~$\lim_{n\to\infty} x^n$~~ in this case also the radius of convergence of $x^n < n!$ (as large n)

then

$$\lim_{n\to\infty} \frac{x^n}{n!} = 0$$

So $\lim_{n\to\infty} \frac{x^n}{n!} = 0$