(a) limit at $x=0$

Choose a path $y = mx$, $\forall m \in \mathbb{R}$

then $f(x, mx) = \{1 - \cos(\frac{x}{m})\}(\sqrt{1+m^2})\cdot x$

at $y = mx$

$$\lim_{x \to 0^+} f(x, mx) = \lim_{h \to 0^+} f(0+h, m(0+h))$$

$$= \lim_{h \to 0} \left(1 - \cos\left(\frac{h}{m}\right)\right)\left(\sqrt{1+m^2}\right)\cdot h$$

$$= 0$$

$$\lim_{x \to 0^-} f(x, mx) = \lim_{h \to 0^-} f(0-h, m(0-h))$$

$$= \lim_{h \to 0} \left(1 - \cos\left(\frac{-h}{m}\right)\right)\left(\sqrt{1+m^2}\right)\cdot(-h)$$

$$= 0$$

$$\lim_{\substack{x \to 0 \\ y \to 0}} f(x,y) = 0 = f(0,0)$$

So f is continuous at $(0,0)$

(d) $\because$ $-1 \leq \cos\left(\frac{x^2}{y}\right) \leq 1$

$$1 \geq -\cos^2\left(\frac{x^2}{y}\right) \geq -1$$

So $1 + 1 \geq 1 - \cos^2\left(\frac{x^2}{y}\right) \geq 1 - 1$

$$0 \le 1 - \cos\left(\frac{x^2}{y}\right) \le 2$$

$$0 \cdot \sqrt{x^2+y^2} \le \left(1 - \cos\left(\frac{x^2}{y}\right)\right) \cdot \sqrt{x^2+y^2}\,) \le 2\sqrt{x^2+y^2}$$

$$0 \le f(x,y) \le \underline{\underline{2\sqrt{x^2+y^2}}}$$

$$\cancel{0 \le (1-\cos}$$

Unbounded