$$I = \int_0^a dx \int_0^{\sqrt{a^2-x^2}} \log(1+x^2+y^2)\, dy$$

$$I = \int_{x=0}^{x=a} \int_{y=0}^{y=\sqrt{a^2-x^2}} \log(1+x^2+y^2)\, dx\, dy$$

$\because$ $y = \sqrt{a^2-x^2} \Rightarrow y^2 = a^2 - x^2$

$\Rightarrow x^2+y^2 = a^2$

Now we change it into polar coordinate with the help $x = r\cos\theta$, $y = r\sin\theta$

$\therefore$ $dx\,dy \to r\,dr\,d\theta$

limits
when

$x = 0 \Rightarrow r\cos\theta = 0$

$\Rightarrow \theta = 0$

$x^2+y^2 = a^2$

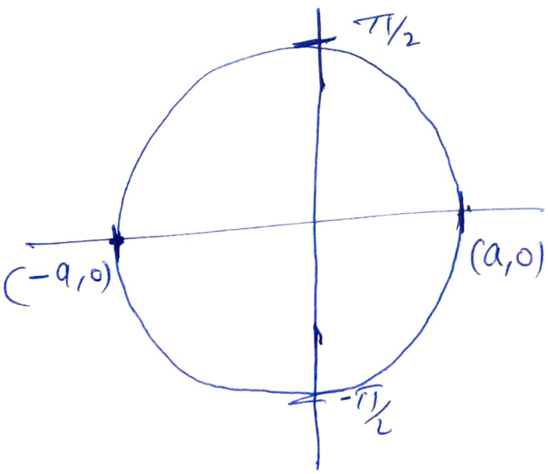
$r^2\cos^2\theta + r^2\sin^2\theta = a^2$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = a^2$

$\Rightarrow r^2 = a^2$

$\Rightarrow \underline{r = \pm a}$

$x =$



$$I = \int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=a} \log(1+r^2)\, r\, dr\, d\theta$$

$$= \int_{r=0}^{r=a} \log(1+r^2)\, r\, dr \left\{ \int_{\theta=0}^{\theta=\pi/2} d\theta \right\}$$

$$I = \int_0^a \log(1+r^2) \cdot r\, dr \cdot (\theta)_{\theta=0}^{\theta=\pi/2}$$

$$= \frac{\pi}{2} \int_0^a \log(1+r^2) \cdot r\, dr$$

put $1+r^2 = t \Rightarrow 2r\, dr = dt$

$\Rightarrow r\, dr = \frac{1}{2} dt$;.

when $r=0$; $t = 1$

$r = a$; $t = 1+a^2$

$$= \frac{\pi}{2} \int_{t=1}^{t=1+a^2} \log t\, dt$$

$$= \frac{\pi}{2} \times \frac{1}{2} \int_{t=1}^{t=1+a^2} 1 \cdot \log t\, dt$$

$$= \frac{\pi}{4} \left[\log t \int 1 \cdot dt - \int \left(\frac{d}{dt}(\log t) \cdot \int dt \right) dt \right]_1^{1+a^2}$$

$$= \frac{\pi}{4} \left[\log t \cdot t - \int \frac{1}{t} \cdot 1 \cdot t\, dt \right]_1^{1+a^2}$$

$$I = \frac{\pi}{4} \left[t \log t - \int dt \right]_1^{1+a^2}$$

$$I = \frac{\pi}{4} \left[(1+a^2)\log(1+a^2) - (1+a^2) - \log 1 + 1 \right]$$

$$I = \frac{\pi}{4} \left[(1+a^2)\log(1+a^2) - a^2 \right]$$