

$$\frac{dy}{dx} + y = f(x)$$

① when $0 \leq x < 1$ then $\frac{dy}{dx} + y = 2$

$$I.F = e^{\int 1 dx} = e^x$$

$$y \cdot e^x = \int 2 \cdot e^x dx + C \Rightarrow y e^x = 2e^x + C$$

$\because y(0) = 0$ so $0 \cdot e^0 = 2 \cdot e^0 + C \Rightarrow 0 = 2 + C$

$$\underline{C = -2}$$

hence

$$y \cdot e^x = 2e^x - 2$$

$$y(x) = 2 - 2e^{-x}$$

$$\underline{y(x) = 2(1 - e^{-x})} ; \text{ when } \underline{0 \leq x < 1}$$

When $x > 1$ then $f(x) = 0$

so $\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -y$

$$\frac{dy}{y} = -dx$$

$$\ln y = -x + \ln C \Rightarrow \ln(y) - \ln C = -x$$

$$\frac{y(x)}{C} = e^{-x} \Rightarrow y(x) = C e^{-x}$$

$$\therefore y(0) = 0$$

$$0 = C - e^0 \Rightarrow C = 0$$

So

$$\underline{y(x) = 0}$$