Given that

$$A \cdot A = \begin{bmatrix} (a_{11}^2 + a_{12}^2 + \cdots a_{1n}^2) & & \\ & (a_{21}^2 + a_{22}^2 + \cdots a_{2n}^2) & \\ & & \ddots \\ & & & a_{n1}^2 + \cdots a_{nn}^2 \end{bmatrix}$$

$$\text{tra}(A \cdot A) = (a_{11}^2 + a_{12}^2 + \cdots a_{1n}^2) + (a_{21}^2 + a_{22}^2 + \cdots a_{2n}^2) + \cdots$$

$$a_{n1}^2 + a_{n2}^2 + \cdots a_{nn}^2$$

(a) If A is invertable $\Rightarrow tr(AA) \neq 0$

$\Rightarrow$ Non zero eigen value

if eigen value of A are $d_1, d_2 \cdots d_n$

$$\Rightarrow tr(A \cdot A) = d_1^2 + d_2^2 + \cdots d_n^2$$

Since all d_i are Non zero

$$\Rightarrow d_1^2 + d_2^2 + \cdots d_n^2 \neq 0$$

(b) ~~Since~~ If tra$(AA) \neq 0$

$$\Rightarrow d_1^2 + d_2^2 + \cdots d_n^2 \neq 0$$

Hence $d_1^2 + d_2^2 + \cdots d_n^2 = 0$ iff all $d_i = 0$

if all $d_i = 0$ then tra$(AA) = 0$ this is contradication

Hence No any $d_i = 0 \Rightarrow A$ is invertable

d) If trace $(AA) = 0$

$\Rightarrow$ $(a_{11}^2 + a_{12}^2 + a_{13}^2 + \cdots a_{1n}^2) + \cdots (a_{n1}^2 + a_{n2}^2 + \cdots a_{nn}^2) = 0$

this ~~is~~ is possible iff all a_{ij} is zero

$\Rightarrow$ A is zero Matrix.

c) $\rightarrow$ option (c) is Not ~~s~~ Clear.