Problem:

$$\int \frac{1}{2\sin^2(x)+3\cos^2(x)}\,\mathrm{d}x$$

Prepare for substitution, use:

$$\sin(x)=\frac{\tan(x)}{\sec(x)},$$

$$\cos(x)=\frac{1}{\sec(x)},$$

$$\sec^2(x)=\tan^2(x)+1$$

$$=\int \sec^2(x)\cdot\frac{1}{2\tan^2(x)+3}\,\mathrm{d}x$$

Substitute $u=\tan(x) \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=\sec^2(x)$ (steps)

$$\longrightarrow \mathrm{d}x=\frac{1}{\sec^2(x)}\,\mathrm{d}u:$$

$$=\int \frac{1}{2u^2+3}\,\mathrm{d}u$$

... or choose an alternative:

Substitute $\cot(x)$

Substitute $v=\frac{\sqrt{2}u}{\sqrt{3}} \longrightarrow \frac{\mathrm{d}v}{\mathrm{d}u}=\frac{\sqrt{2}}{\sqrt{3}}$ (steps) $\longrightarrow$

$$\mathrm{d}u=\frac{\sqrt{3}}{\sqrt{2}}\,\mathrm{d}v:$$

$$=\int \frac{\sqrt{3}}{\sqrt{2}\,(3v^2+3)}\,\mathrm{d}v$$

Simplify:

$$=\frac{1}{\sqrt{2}\sqrt{3}}\int \frac{1}{v^2+1}\,\mathrm{d}v$$

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Now solving:

$$\int \frac{1}{v^2+1}\,dv$$

This is a standard integral:

$$= \arctan(v)$$

Plug in solved integrals:

$$\frac{1}{\sqrt{2}\sqrt{3}}\int \frac{1}{v^2+1}\,dv$$

$$= \frac{\arctan(v)}{\sqrt{2}\sqrt{3}}$$

Undo substitution $v = \frac{\sqrt{2}u}{\sqrt{3}}$:

$$= \frac{\arctan\left(\frac{\sqrt{2}u}{\sqrt{3}}\right)}{\sqrt{2}\sqrt{3}}$$

Undo substitution $u = \tan(x)$:

$$= \frac{\arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{3}}\right)}{\sqrt{2}\sqrt{3}}$$

The problem is solved:

$$\int \frac{1}{2\sin^2(x)+3\cos^2(x)}\,dx$$

$$= \frac{\arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{3}}\right)}{\sqrt{2}\sqrt{3}} + C$$

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