

Representation of $\phi(n)$

$\therefore$ Group $U(n) \cong \mathbb{Z}_{\phi(n)}$

$$U(n) = \{x \mid \gcd(x, n) = 1\}$$

$$U(4) = \{1, 3\}$$

$$\therefore U(4) \cong \mathbb{Z}_{\phi(4)} = \mathbb{Z}_2$$

$$U(5) = \{1, 2, 3, 4\}$$

$$U(5) \cong \mathbb{Z}_4$$

$$U(6) = \{1, 5\}$$

$$U(6) \cong \mathbb{Z}_3$$

$$U(8) = \{1, 3, 5, 7\}$$

all are cyclic

$$U(8) \cong \mathbb{Z}_{\phi(8)}$$

$$\therefore 1^4 = 1 \mid O(1) = 1$$

$$3^4 = 3$$

$$3^2 = 9 \equiv 1 \pmod{8}$$

$$O(3) = 2$$

$$5^4 = 5$$

$$5^2 = 25 \equiv 1 \pmod{8}$$

$$\therefore 7^4 = 7$$

$$7^2 = 49 \equiv 1 \pmod{8}$$

$$O(7) = 2$$

So in $U(8)$ if any element which order is equal to order of the group
i.e. if any element of order 4