$$\sum_{n=1}^{\infty} \left[\frac{1}{n} - \sin\left(\frac{1}{n}\right)\right]^{\alpha}$$

$\because$ Expansion of $\sin\left(\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{3!n^3} + \frac{1}{5!n^5} - \frac{1}{7!n^7} + \frac{1}{9!n^9} - \cdots$

$$= \sum_{n=1}^{\infty} \left[\frac{1}{n} - \left(\frac{1}{n} - \frac{1}{3!n^3} + \frac{1}{5!n^5} - \frac{1}{7!n^7} + \frac{1}{9!n^9} - \frac{1}{11!n^{11}} + \cdots\right)\right]^{\alpha}$$

$$= \sum_{n=1}^{\infty} \left[\cancel{\frac{1}{n}} - \cancel{\frac{1}{n}} + \frac{1}{3!n^3} - \frac{1}{5!n^5} + \frac{1}{7!n^7} - \frac{1}{9!n^9} + \frac{1}{11!n^{11}} - \cdots\right]^{\alpha}$$

$$\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} \frac{1}{n^{3\alpha}}\left[\frac{1}{3!} - \frac{1}{5!n^2} + \frac{1}{7!n^4} - \cdots\right]^{\alpha}$$

Now let an auxiliary equation $\sum v_n = \sum \frac{1}{n^{3\alpha}}$ such that by limit comparision test

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{\cancel{\frac{1}{n^{3\alpha}}}\left[\frac{1}{3!} - \frac{1}{5!n^2} + \cdots\right]}{\frac{1}{n^{3\alpha}}}$$

$$= \frac{1}{3!}$$

Hence Both series will be either convergent or divergent simultaneously, so

Now $\sum v_n = \sum \frac{1}{n^{3\alpha}}$ will be convergent if

$3\alpha > 1$

$\alpha > \frac{1}{3}$

$\because$ By p test

$\sum \frac{1}{n^p}$ is convergent if $p > 1$

divergent if $p \leq 1$

So $\sum \left[\frac{1}{n} - \sin\left(\frac{1}{n}\right)\right]^{\alpha}$ will convergent for $\alpha > \frac{1}{3}$

option A is correct

$$\sum_{n=1}^{\infty}\left[\frac{1}{n}-\sin\left(\frac{1}{n}\right)\right]^{\alpha}$$

$\because$ Expansion of $\sin\left(\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{3!\,n^3} + \frac{1}{5!\,n^5} - \frac{1}{7!\,n^7} + \frac{1}{9!\,n^9} - \cdots$

$$= \sum_{n=1}^{\infty}\left[\frac{1}{n} - \left(\frac{1}{n} - \frac{1}{3!\,n^3} + \frac{1}{5!\,n^5} - \frac{1}{7!\,n^7} + \frac{1}{9!\,n^9} - \frac{1}{11!\,n^{11}} + \cdots\right)\right]^{\alpha}$$

$$= \sum_{n=1}^{\infty}\left[\cancel{\frac{1}{n}} - \cancel{\frac{1}{n}} + \frac{1}{3!\,n^3} - \frac{1}{5!\,n^5} + \frac{1}{7!\,n^7} - \frac{1}{9!\,n^9} + \frac{1}{11!\,n^{11}} - \cdots\right]^{\alpha}$$

$$\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} \frac{1}{n^{3\alpha}}\left[\frac{1}{3!} - \frac{1}{5!\,n^2} + \frac{1}{7!\,n^4} - \cdots\right]^{\alpha}$$

Now let an auxiliary equation $\sum v_n = \sum \frac{1}{n^{3\alpha}}$ such that by limit comparision test

$$\lim_{n\to\infty}\frac{u_n}{v_n} = \lim_{n\to\infty}\frac{\cancel{\frac{1}{n^{3\alpha}}}\left[\frac{1}{3!} - \frac{1}{5!\,n^2} + \cdots\right]}{\frac{1}{n^{3\alpha}}}$$

$$= \frac{1}{3!}$$

hence Both series will be either convergent or divergent simultaneously, so

Now $\sum v_n = \sum \frac{1}{n^{3\alpha}}$ will be convergent if

$3\alpha > 1$

$\alpha > \frac{1}{3}$

$\because$ By p test

$\sum \frac{1}{n^p}$ is convergent if $p > 1$

divergent if $p \le 1$

So $\sum\left[\frac{1}{n} - \sin\left(\frac{1}{n}\right)\right]^{\alpha}$ will convergent for $\alpha > \frac{1}{3}$

option A is correct