Problem:

$$\int \frac{x^2}{(bx+a)^2}\,\mathrm{d}x$$

Substitute $u = bx + a \longrightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = b$ (steps) $\longrightarrow$

$\mathrm{d}x = \dfrac{1}{b}\,\mathrm{d}u$, use:

$$x^2 = \frac{(u-a)^2}{b^2}$$

$$= \frac{1}{b^3}\int \frac{(u-a)^2}{u^2}\,\mathrm{d}u$$

... or choose an alternative:

Substitute $\dfrac{x}{bx+a}$

Substitute $\dfrac{bx+a}{x}$

Don't substitute

Now solving:

$$\int \frac{(u-a)^2}{u^2}\,\mathrm{d}u$$

Expand:

$$= \int \left(-\frac{2a}{u} + \frac{a^2}{u^2} + 1\right)\mathrm{d}u$$

Apply linearity:

$$= -2a \cdot \int \frac{1}{u}\,\mathrm{d}u + a^2 \int \frac{1}{u^2}\,\mathrm{d}u + \int 1\,\mathrm{d}u$$

Now solving:

$$\int \frac{1}{u}\,\mathrm{d}u$$

This is a standard integral:

$$= \ln(u)$$

Now solving:

$$\int \frac{1}{u^2}\,\mathrm{d}u$$

Apply power rule:

$$\int u^{\mathrm{n}}\,\mathrm{d}u = \frac{u^{\mathrm{n}+1}}{\mathrm{n}+1} \text{ with } \mathrm{n} = -2:$$

$$= -\frac{1}{u}$$

Now solving:

$$\int 1\,\mathrm{d}u$$

Apply constant rule:

$$= u$$

Plug in solved integrals:

$$-2a \cdot \int \frac{1}{u}\,\mathrm{d}u + a^2 \int \frac{1}{u^2}\,\mathrm{d}u + \int 1\,\mathrm{d}u$$

$$= -2a\ln(u) + u - \frac{a^2}{u}$$

Plug in solved integrals:

$$\frac{1}{b^3}\int \frac{(u-a)^2}{u^2}\,\mathrm{d}u$$

$$= -\frac{2a\ln(u)}{b^3} + \frac{u}{b^3} - \frac{a^2}{b^3 u}$$

Undo substitution $u = bx + a$:

$$= -\frac{2a\ln(bx+a)}{b^3} - \frac{a^2}{b^3(bx+a)} + \frac{bx+a}{b^3}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{x^2}{(bx+a)^2}\,\mathrm{d}x$$

$$= -\frac{2a\ln(|bx+a|)}{b^3} - \frac{a^2}{b^3(bx+a)} + \frac{bx+a}{b^3} + C$$

Rewrite/simplify:

$$= \frac{bx - 2a\ln(|bx+a|)}{b^3} - \frac{a^2}{b^3(bx+a)} + C$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$\int f(x)\,\mathrm{d}x = F(x) =$

$$-\frac{2a\ln(|bx+a|)}{b^3} - \frac{a^2}{b^4x + ab^3} + \frac{x}{b^2} + C$$