Problem:

$$\int e^{\sqrt{x}}\,dx$$

Substitute $u = \sqrt{x} \longrightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$ (steps) $\longrightarrow$

$dx = 2\sqrt{x}\,du$:

$$= 2\int u e^{u}\,du$$

... or choose an alternative:

Substitute $e^{\sqrt{x}}$

Now solving:

$$\int u e^{u}\,du$$

Integrate by parts: $\int f g' = f g - \int f' g$

$f = u, \quad g' = e^{u}$

$\downarrow$ steps $\quad \downarrow$ steps

$f' = 1, \quad g = e^{u}$:

$$= u e^{u} - \int e^{u}\,du$$

Now solving:

$$\int e^{u}\,du$$

Apply exponential rule:

$\int a^{u}\,du = \frac{a^{u}}{\ln(a)}$ with $a = e$:

$$= e^{u}$$

Plug in solved integrals:

$$ue^u - \int e^u \, du$$

$$= ue^u - e^u$$

Plug in solved integrals:

$$2\int ue^u \, du$$

$$= 2ue^u - 2e^u$$

Undo substitution $u = \sqrt{x}$:

$$= 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}$$

The problem is solved:

$$\int e^{\sqrt{x}} \, dx$$

$$= 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

Rewrite/simplify:

$$= 2\left(\sqrt{x} - 1\right)e^{\sqrt{x}} + C$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$$\int \boldsymbol{f(x)}\,\mathbf{d}\boldsymbol{x} = \boldsymbol{F(x)} =$$

$$2\left(\sqrt{x} - 1\right)e^{\sqrt{x}} + C$$