Problem:

$$\int x\sqrt{4x+3}\,\mathrm{d}x$$

Substitute $u = 4x + 3 \longrightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 4$ (steps) $\longrightarrow$

$\mathrm{d}x = \dfrac{1}{4}\,\mathrm{d}u$:

$$= \frac{1}{16}\int\left(u^{\frac{3}{2}} - 3\sqrt{u}\right)\mathrm{d}u$$

Now solving:

$$\int\left(u^{\frac{3}{2}} - 3\sqrt{u}\right)\mathrm{d}u$$

Apply linearity:

$$= \int u^{\frac{3}{2}}\,\mathrm{d}u - 3\int\sqrt{u}\,\mathrm{d}u$$

Now solving:

$$\int u^{\frac{3}{2}}\,\mathrm{d}u$$

Apply power rule:

$\displaystyle\int u^{\mathrm{n}}\,\mathrm{d}u = \frac{u^{\mathrm{n}+1}}{\mathrm{n}+1}$ with $\mathrm{n} = \dfrac{3}{2}$:

$$= \frac{2u^{\frac{5}{2}}}{5}$$

Now solving:

$$\int \sqrt{u}\,du$$

Apply power rule with $\mathbf{n} = \frac{1}{2}$:

$$= \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\int u^{\frac{3}{2}}\,du - 3\int \sqrt{u}\,du$$

$$= \frac{2u^{\frac{5}{2}}}{5} - 2u^{\frac{3}{2}}$$

Plug in solved integrals:

$$\frac{1}{16}\int \left(u^{\frac{3}{2}} - 3\sqrt{u}\right) du$$

$$= \frac{u^{\frac{5}{2}}}{40} - \frac{u^{\frac{3}{2}}}{8}$$

Undo substitution $u = 4x + 3$:

$$= \frac{(4x+3)^{\frac{5}{2}}}{40} - \frac{(4x+3)^{\frac{3}{2}}}{8}$$

The problem is solved:

$$\int x\sqrt{4x+3}\,\mathrm{d}x$$

$$=\frac{(4x+3)^{\frac{5}{2}}}{40}-\frac{(4x+3)^{\frac{3}{2}}}{8}+C$$

Rewrite/simplify:

$$=\frac{(4x-2)\,(4x+3)^{\frac{3}{2}}}{40}+C$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$$\int f(x)\,\mathrm{d}x=F(x)=$$

$$\frac{(4x+3)^{\frac{5}{2}}}{40}-\frac{(4x+3)^{\frac{3}{2}}}{8}+C$$

Simplify/rewrite:

$$\frac{(2x-1)\,(4x+3)^{\frac{3}{2}}}{20}+C$$

Simplify