$\because$ $f : [-1, 1] \to R$ is continuous function such that

$$\int_{-1}^{1} f(t)\,dt = 2$$

let $f(x) = K$; $\forall x \in \mathbb{R}$ and K be any fixed real number

then $\int_{-1}^{1} f(t)\,dt = 2 \Rightarrow \int_{-1}^{1} K\,dt = 2$

$$\Rightarrow K\,(t)\Big|_{-1}^{1} = 2$$

$$\Rightarrow K(2) = 2$$

$$\Rightarrow \underline{K = 1}$$

so

$f(x) = 1$; $\forall x \in \mathbb{R}$.

Now $\lim_{n\to\infty} \int_{-1}^{1} 1 \cdot \sin^2(nt)\,dt = \lim_{n\to\infty} \int_{-1}^{1} \frac{1-\cos(2nt)}{2}\,dt$

$$= \frac{1}{2}\left[\lim_{n\to\infty}\left(\int_{-1}^{1} dt - \int_{-1}^{1}\cos(2nt)\,dt\right)\right]$$

$$= \frac{1}{2}\left[\lim_{n\to\infty}\left(2 - \left(\frac{\sin 2nt}{2n}\Big|_{-1}^{1}\right)\right)\right]$$

$$= \frac{1}{2}\left[2 - \left(\frac{\sin(2n)}{2n} + \frac{\sin 2n}{2}\right.\right.$$

$$-1 \leq \sin 2n \leq 1$$

$$\frac{-1}{2n} \leq \frac{\sin 2n}{2n} \leq \frac{1}{2n}$$

$$\lim_{n\to\infty} \frac{-1}{2n} \leq \lim_{n\to\infty} \frac{\sin(2n)}{2n} \leq \lim_{n\to\infty} \frac{1}{2n}$$

$$0 \leq \lim_{n\to\infty} \frac{\sin(2n)}{2n} \leq 0$$

$$\Rightarrow \frac{1}{2}(2-0)$$

$$= 1$$