$$x \frac{d}{dx}\left(\underset{I}{x}\underset{II}{\frac{dy}{dx}} - y\right) - 2x\frac{dy}{dx} + 2y + x^2 y = 0 \quad \text{---(1)}$$

Now

$$x\frac{d}{dx}\left(x\frac{dy}{dx} - y\right) = x\left[\frac{d}{dx}\left(x\frac{dy}{dx}\right) - \frac{d}{dx}(y)\right]$$

$$\because \quad \frac{d}{dx}[f+g] = \frac{df}{dx} + \frac{dg}{dx}$$

$$= x\left[\frac{d}{dx}\left(\underset{I}{x}\underset{II}{\frac{dy}{dx}}\right) - \frac{dy}{dx}\right]$$

$$= x\left[\frac{d}{dx}(x)\cdot\frac{dy}{dx} + x\frac{d}{dx}\left(\frac{dy}{dx}\right) - \frac{dy}{dx}\right]$$

$$= x\left[1\cdot\frac{dy}{dx} + x\frac{d^2y}{dx^2} - \frac{dy}{dx}\right]$$

$$= x\left(x\frac{d^2y}{dx^2}\right)$$

$$= x^2\frac{d^2y}{dx^2} \quad \text{---(2)}$$

from eqn (1) & (2)

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (2+x^2)y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(\frac{2}{x^2}+1\right)y = 0$$