$$\frac{1}{2(2^2-1)} + \frac{1}{3(3^2-1)} + \frac{1}{4(4^2-1)} + \frac{1}{5(5^2-1)} + \cdots$$

$\because$ we know $a^2-b^2 = (a-b)(a+b)$

so $(2^2-1) = (2-1)(2+1) = 1\cdot 3$

$(3^2-1) = (3-1)(3+1) = 2\cdot 4$

$(4^2-1) = (4-1)(4+1) = 3\cdot 5$

$(5^2-1) = (5-1)(5+1) = 4\cdot 6$

$\vdots$

so

$$\frac{1}{2(2^2-1)} + \frac{1}{3(3^2-1)} + \frac{1}{4(4^2-1)} + \frac{1}{5(5^2-1)} + \cdots$$

converts in this following mannew

$$\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \frac{1}{3\cdot 4\cdot 5} + \frac{1}{4\cdot 5\cdot 6} + \cdots$$

$$\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$$

$$U_n = \frac{1}{n(n+1)(n+2)}$$

from the partial fraction theory

$$\frac{1}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2}$$

Page No.

Date: / /

$$A = \frac{1}{(0+1)(0+2)} = \frac{1}{2}$$

$$B = \frac{1}{(-1)(-1+2)} = -1 \quad ; \quad C = \frac{1}{(-2)(-2+1)} = \frac{1}{2}$$

$A = \frac{1}{2}$; $B = -1$; $C = \frac{1}{2}$

hence

$$U_n = \frac{1/2}{n} + \frac{-1}{n+1} + \frac{1/2}{n+2}$$

$$U_n = \frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2(n+2)}$$

$$U_1 = \frac{1}{2} - \frac{1}{2} + \frac{1}{2 \cdot 3}$$

$$U_2 = \frac{1}{2 \cdot 2} - \frac{1}{3} + \frac{1}{2 \cdot 4}$$

$$U_3 = \frac{1}{2 \cdot 3} - \frac{1}{4} + \frac{1}{2 \cdot 5}$$

$$U_4 = \frac{1}{2 \cdot 4} - \frac{1}{5} + \frac{1}{2 \cdot 6}$$

$\vdots$

$$U_{k-3} = \frac{1}{2(k-3)} - \frac{1}{(k-2)} + \frac{1}{2(k-1)}$$

$$U_{k-2} = \frac{1}{2(k-2)} - \frac{1}{(k-1)} + \frac{1}{2 \cdot k}$$

$$U_{k-1} = \frac{1}{2k-1} - \frac{1}{k} + \frac{1}{2(k+1)}$$

$$U_k = \frac{1}{2k} - \frac{1}{k+1} + \frac{1}{2(k+2)}$$

Now on adding ~~and~~ these terms we get

$$\sum_{n=1}^{k} U_n = \frac{1}{2} - \frac{1}{2} + \frac{1}{4} + \frac{1}{2(k+1)} - \frac{1}{k+1} + \frac{1}{2(k+2)}$$

$$\lim_{k \to \infty} \sum_{n=1}^{k} U_n = \sum_{n=1}^{\infty} U_n = \lim_{k \to \infty} \left[\frac{1}{4} - \frac{1}{2(k+1)} + \frac{1}{2(k+2)} \right]$$

$$= \frac{1}{4} - 0 + 0$$

$$= \frac{1}{4}$$ Ans