$$f(x) = \frac{1}{x} \qquad x \in S$$

S contains element of type $\frac{1}{n}$

So, $\boxed{f\left(\frac{1}{x}\right) = x}$ — (i)

at $x = 3$

$$\boxed{f\left(\frac{1}{3}\right) = 3}$$

Consider

$$\left| f(x) - f\left(\frac{1}{3}\right) \right| < 1$$

$$|f(x) - 3| < 1$$

$$\boxed{2 < f(x) < 4}$$

$$\Rightarrow f^{-1}(2) < x < f^{-1}(4)$$

$$\Rightarrow \boxed{\frac{1}{2} < x < \frac{1}{4}} \qquad \text{from (i)} \quad - (3)$$

if $\left| x - \frac{1}{3} \right| < \delta$

$$\boxed{\frac{1}{3} - \delta < x < \frac{1}{3} + \delta} \quad - (2)$$

Compare (2) & (3)

$$\frac{1}{3} - \delta = \frac{1}{2} \quad \& \quad \frac{1}{3} + \delta = \frac{1}{4}$$

$$\delta = \frac{1}{6} \quad \& \quad \delta = \frac{1}{12}$$

$$\max\left\{\frac{1}{6}, \frac{1}{12}\right\} = \boxed{\frac{1}{6}} \text{ or } \boxed{0.167}$$