

$$\text{let } y = \frac{1}{n} [(a+1) \cdots (a+n)]^{\frac{1}{n}}$$

$$y = \left[\frac{(a+1)(a+2) \cdots (a+n)}{n^n} \right]^{\frac{1}{n}}$$

here let $a_n = \frac{(a+1)(a+2) \cdots (a+n)}{n^n}$

then

$$\frac{a_{n+1}}{a_n} = \frac{(a+1)(a+2) \cdots (a+n)(a+n+1)}{(n+1)^{n+1}} \times \frac{n^n}{(a+1) \cdots (a+n)}$$

$$= \frac{a+n+1}{\left(1+\frac{1}{n}\right)^n (1+n)} = \frac{\frac{a+1}{n} + 1}{\left(1+\frac{1}{n}\right)^{n+1}}$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{0+1}{e \cdot 1} = \frac{1}{e}$$

Thus Cauchy's second theorem on limit

$$\lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}} = \frac{1}{e}$$

$S_n, \boxed{x=1} R$