Q $\sum_{n=1}^{\infty} \frac{2n}{3^n} \Rightarrow u_n = \frac{2n}{3^n}$

by ratio test $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{2n+2}{3^{n+1}} \times \frac{3^n}{2n}$

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{\cancel{2n}(1+\frac{1}{n})}{3 \cdot \cancel{3^n}} \times \frac{\cancel{3^n}}{\cancel{2n}}$

$= \frac{1}{3} \neq 0$

$= \frac{1}{3} < 1$

$\Rightarrow \sum u_n$ is convergent

(11)

$\sum \frac{2}{1^2} + \frac{3}{2^2} + \frac{4}{3^2} + \frac{5}{4^2} + \cdots$

this series can be written as below

$\sum u_n = \sum_{n=1}^{\infty} \frac{(n+1)}{n^2} = \sum_{n=1}^{\infty} \frac{n}{n^2} + \sum_{n=1}^{\infty} \frac{1}{n^2}$

$= \sum_{n=1}^{\infty} \frac{1}{n} + \sum_{n=1}^{\infty} \frac{1}{n^2}$

By P - test $\sum \frac{1}{n^p}$ is convergent if $p > 1$

is divergent if $p \leq 1$

So $\sum \frac{1}{n}$ is divergent $\because p=1$

$\sum \frac{1}{n^2}$ is convergent $\because p>1$

So $\sum u_n = \sum \frac{1}{n} + \sum \frac{1}{n^2}$

is divergent bcoz sum of a convergent and divergent series is a divergent

cgt + dgt = dgt