① $\sum \dfrac{2^{n-1}}{3^{n+1}}$

Use ratio test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{2^n}{3^{n+2}} \times \frac{3^{n+1}}{2^{n-1}}\right| = \frac{2}{3} < 1$$

$\Rightarrow$ series is convergent

② $\sum \dfrac{n!}{(2)^{2n-1}}$

Use ratio test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!}{2^{2n+1}} \times \frac{2^{2n-1}}{n!}\right| = \lim \left|\frac{n+1}{2^n \cdot 2^{2n+1}}\right| \Rightarrow 0 < 1$$

$\Rightarrow$ convergent

③ $\displaystyle\sum_{n=0}^{\infty} (n+1)^2 x^n$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+2)^2 x^{n+1}}{(n+1)^2 x^n}\right| = \lim_{n\to\infty}\left|\left(\frac{n+2}{n+1}\right)^2 x\right|$$

$$\lim_{n\to\infty}\left|\left(\frac{1+\frac{2}{n}}{1+\frac{1}{n}}\right)^2 x\right| = |x|$$

convergent for $|x|<1$
divergent for $|x|>1$

at $x=1$ $\quad \sum (n+1)^2 =$ divergent

at $x=-1$ $\quad \sum (-1)^n (n+1)^2 \to$ divergent

Thus interval of convergence $= (-1, 1)$