

$$\lim_{x \rightarrow 0} \frac{\tan 2x - 2 \sin x}{x(1 - \cos 2x)} = \beta$$

expansion of $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \dots$

$$\tan 2x = 2x + \frac{(2x)^3}{3} + \frac{2}{15}(2x)^5 + \dots$$

Expansion of $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

$$\sin 2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \dots$$

Exp.

$$\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \dots$$

Now

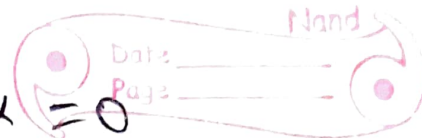
$$\lim_{x \rightarrow 0} \frac{\left(2x + \frac{(2x)^3}{3} + \frac{2}{15}(2x)^5 + \dots \right) - 2 \left(2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \dots \right)}{x \left(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \dots \right)}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{(2 - 2x)x + \left(\frac{8}{3} + \frac{2x^3}{3!} \right)x^3 + \left(\frac{2^6}{15} - \frac{2x^5}{5!} \right)x^5 + \dots}{\left(\frac{2^2 x^3}{2!} + \frac{2}{4!}x^5 + \dots \right)}$$

Now here the denominator has more power of x so limit exist

finitely if $\alpha = \frac{1}{2}$

So if $\alpha = \frac{1}{2}$ then $2 - 2\alpha = 0$



So take $\alpha = 1$

$$\lim_{x \rightarrow 0} 0 \cdot x + \left(\frac{8}{3} + \frac{2}{3!} \right) x^3 + \left(\frac{2^6}{18} - \frac{2}{5!} \right) x^5 + \dots$$

$$\frac{2x^3}{2!} + \frac{2x^5}{4!} + \dots$$

$$= \frac{\left(\frac{8}{3} + \frac{2}{6} \right) + 0 + 0 + \dots}{9 - 0 + 0 - \dots}$$

$$= \frac{9}{3 \times 2} = \frac{3}{2}$$

$$\beta = \frac{3}{2}$$

~~Hence $\alpha + \beta = 1 + 3 = 4$~~

$$\alpha + \beta = 1 + \frac{3}{2} = \frac{5}{2} \text{ Ans}$$