$$\lim_{x \to 0} \frac{\tan 2x - 2\sin \alpha x}{x(1-\cos 2x)} = \beta$$

expansion of $\tan x = x + \frac{x^3}{3} + \frac{2}{15} x^5 + \cdots$

$$\tan 2x = 2x + \frac{(2x)^3}{3} + \frac{2}{15}(2x)^5 + \cdots$$

Expansion of $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$$\sin \alpha x = \alpha x - \frac{(\alpha x)^3}{3!} + \frac{(\alpha x)^5}{5!} + \cdots$$

~~Exp.~~

$$\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \cdots$$

Now

$$\lim_{x \to 0} \frac{\left(2x + \frac{(2x)^3}{3} + \frac{2}{15}(2x)^5 + \cdots\right) - 2\left(\alpha x - \frac{(\alpha x)^3}{3!} + \frac{(\alpha x)^5}{5!} + \cdots\right)}{x\left(\cancel{1} - \cancel{1} + \frac{(2x)^2}{2!} - \frac{(2x)^4}{4!} + \cdots\right)}$$

$$\Rightarrow \lim_{x \to 0} \frac{(2-2\alpha)x + \left(\frac{8}{3} + \frac{2\alpha^3}{3!}\right)x^3 + \left(\frac{2^6}{15} - \frac{2\alpha^5}{5!}\right)x^5 + \cdots}{\left(\frac{2^2 x^3}{2!} - \frac{2}{4!}x^5 + \cdots\right)}$$

Now here the denomenator has more power of x so limit exist

Finitely if $\alpha = \frac{1}{1}$

so if $\alpha = 1$ then $2 - 2\alpha = 0$

so take $\alpha = 1$

$$\lim_{x \to 0} \frac{0 \cdot x + \left(\frac{8}{3} + \frac{2}{3!}\right)x^3 + \left(\frac{2^6}{15} - \frac{2}{5!}\right)x^5 + \cdots}{\frac{2^2 x^3}{2!} + \frac{2}{4!}x^5 + \cdots}$$

$$= \frac{\left(\frac{8}{3} + \frac{2}{6}\right) + 0 + 0 + \cdots}{2 - 0 + 0 - \cdots}$$

$$= \frac{9}{3 \times 2} = \frac{3}{2}$$

$\beta = \frac{3}{2}$

~~hence $\alpha + \beta = 1 + 3 = 4$~~

$\alpha + \beta = 1 + \frac{3}{2} = \frac{5}{2}$ Ans