

$$f(x) = \begin{cases} \sin x & ; x = n\pi, n \in \mathbb{Z} \\ 2 & ; \text{otherwise} \end{cases}$$

$$g(x) = \begin{cases} x^2 + 1 & ; x \neq 0, 2 \\ 4 & x \neq 0 \\ 5 & x = 2 \end{cases}$$

$$\lim_{x \rightarrow 0} g\{f(x)\} = ?$$

$$\text{LHL } \lim_{x \rightarrow 0^-} g\{f(x)\} = \lim_{h \rightarrow 0} g\{f(0-h)\}$$

$$= \lim_{h \rightarrow 0} g\{f(-h)\}$$

$$\because f(-h) = 2$$

$$= \lim_{h \rightarrow 0} g\{2\}$$

$$\text{LHL} = 5$$

Now RHL

$$\lim_{x \rightarrow 0^+} g(f(x)) = \lim_{h \rightarrow 0} g(f(0+h))$$

$$= \lim_{h \rightarrow 0} g(f(h))$$

$$\because f(h) = 2$$

$$= \lim_{h \rightarrow 0} g(2)$$

$$\text{RHL} = 5$$

$$\because g(2) = 5$$

$$\text{hence } \text{LHL} = \text{RHL} = 5$$

$$\underline{\text{So}} \quad \lim_{x \rightarrow 0} g(f(x)) = 5$$