

$$(D^2 - 4)y = \sin^2 x$$

C.F

$$D^2 - 4 = 0 \Rightarrow (D+2)(D-2) = 0$$

$$\Rightarrow D = 2, -2$$

$$\text{C.F} = c_1 e^{2x} + c_2 e^{-2x}$$

P.I

$$\frac{\sin^2 x}{(D^2 - 4)} = \frac{1 - \cos 2x}{2(D^2 - 4)} = \frac{1}{2(D^2 - 4)} - \frac{\cos 2x}{2(D^2 - 4)}$$

$$= \frac{e^{0x}}{2(D^2 - 4)} - \frac{\cos 2x}{2(D^2 - 4)}$$

(replace D by 0) (replace D^2 by $-(2)^2$)

$$= \frac{e^{0x}}{2(-4)} - \frac{\cos 2x}{2(-4-4)} \Rightarrow \boxed{-\frac{1}{8} + \frac{\cos 2x}{16}} \rightarrow \underline{\underline{\text{P.I}}}$$

Thus $y = \text{C.F} + \text{P.I}$

$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{8} + \frac{\cos 2x}{16}$$