$$\sum_{n=1}^{\infty} \frac{n}{(2n+1)^2}(x-2)^{3n}$$

Use ratio test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)\cancel{(x-2)}^{3n+3}}{(2n+3)^2} \times \frac{(2n+1)^2}{n\cancel{(x-2)^{3n}}}\right| = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)\cdot\left(\frac{1+\frac{1}{2n}}{1+\frac{3}{2n}}\right)^2 (x-2)^3$$

$$= 1\cdot 1\cdot (x-2)^3 = r$$

Thus convergent if $|r|<1$

$$\Rightarrow |(x-2)^3|<1 \Rightarrow \boxed{-3<x<3}$$

at $x=-3$ $\quad \sum_{n=1}^{\infty}\frac{n}{(2n+1)^2}(-5)^{3n} = \sum \frac{(-1)^n n(5)^{3n}}{(2n+1)^2} \to$ divergent

at $x=3$ $\quad \sum_{n=1}^{\infty}\frac{n}{(2n+1)^2} \to$ convergent

Thus interval of convergence $(-3,3]$

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