Suppose $x_1 < x_2$, Then

$$x_1 < \frac{x_1+x_2}{2} < x_2$$

$\Rightarrow$ Applying mean value theorem to $f$ in interval $\left[x_1, \frac{x_1+x_2}{2}\right]$

$\exists$ a point $\alpha \in \left(x_1, \frac{x_1+x_2}{2}\right)$ such that

$$f'(\alpha) = \frac{f\left(\frac{x_1+x_2}{2}\right) - f(x_1)}{\frac{x_1+x_2}{2} - x_1} = \frac{f\left(\frac{x_1+x_2}{2}\right) - f(x_1)}{\frac{x_2-x_1}{2}} \quad \text{—(i)}$$

Ily by mean value theorem in $\left[\frac{x_1+x_2}{2}, x_2\right]$, $\exists$ a point $\beta \in \left(\frac{x_1+x_2}{2}, x_2\right)$ such that

$$f'(\beta) = \frac{f(x_2) - f\left(\frac{x_1+x_2}{2}\right)}{x_2 - \frac{x_1+x_2}{2}} = \frac{f(x_2) - f\left(\frac{x_1+x_2}{2}\right)}{\frac{x_2-x_1}{2}} \quad \text{—(ii)}$$

$\Rightarrow$ $x_1 < \alpha < \frac{x_1+x_2}{2} < \beta < x_2 \Rightarrow \boxed{\alpha < \beta}$

$\Rightarrow$ Now applying MVT to $f'$ in $[\alpha, \beta]$, $\exists$ a $\gamma \in (a, b)$ such that

$$0 < f''(\gamma) = \frac{f'(\beta) - f'(\alpha)}{\beta - \alpha}$$

$$\Rightarrow \frac{f'(\beta) - f'(\alpha)}{\beta - \alpha} > 0 \Rightarrow \boxed{f'(\beta) - f'(\alpha) > 0}$$

$$\Rightarrow \frac{f(x_2) + f(x_1) - 2f\left(\frac{x_1+x_2}{2}\right)}{\frac{x_2-x_1}{2}} > 0 \quad \text{[from (i) \& (ii)]}$$

$$\Rightarrow f(x_1) + f(x_2) - 2f\left(\frac{x_1+x_2}{2}\right) > 0$$

$$\Rightarrow \boxed{\frac{f(x_1)+f(x_2)}{2} > f\left(\frac{x_1+x_2}{2}\right)}$$