

$$\vec{F} = (2xy+2)\hat{i} + (2+x^2)\hat{j} + (x+y)\hat{k}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy+2 & 2+x^2 & x+y \end{vmatrix}$$

$$= \hat{i}(1-1) - \hat{j}(1-1) + \hat{k}(2x-2x) = \boxed{0}$$

$\Rightarrow F$ is conservative

$\Rightarrow$ a scalar potential exist such that

$$\vec{F} = \nabla \phi$$

$$\frac{\partial \phi_1}{\partial x} = 2xy+2, \quad \frac{\partial \phi_2}{\partial y} = 2+x^2, \quad \frac{\partial \phi_3}{\partial z} = x+y$$

$$\phi_1 = x^2y + 2x$$

$$\phi_2 = zy + x^2y$$

$$\phi_3 = zx + zy$$

Thus $\boxed{\phi = x^2y + 2x + zy}$

$$\int \vec{F} \cdot d\vec{s} = \phi(2,1,-1) - \phi(0,0,0)$$

$$= (4-2-1)-(0) \Rightarrow \boxed{1}$$