Let A be an $n \times n$ matrix and let Λ be an $n \times n$ diagonal matrix. Is it always the case that $A\Lambda = \Lambda A$? If not, when is it the case that $A\Lambda = \Lambda A$?

If we restrict the diagonal entries of Λ to being the equal (i.e. $\Lambda = \mathrm{drag}(a, a, \ldots, a)$), then it is clear that $A\Lambda = AaI = aIA = \Lambda A$. However, I can't seem to come up with an argument for the general case.

It is possible that a diagonal matrix Λ commutes with a matrix A when A is symmetric and $A\Lambda$ is also symmetric. We have

$$\Lambda A = (A^\top \Lambda^\top)^\top = (A\Lambda)^\top = A\Lambda$$

The above trivially holds when A and Λ are both diagonal.

If all the diagonal entries of Λ are distinct, it commutes only with diagonal matrices.

In contrast, for each k consecutive equal diagonal entries in Λ, we may allow A to have anything at all in the corresponding k by k square block with both corners on the main diagonal.

This means that the set of matrices that commute with Λ has a minimum dimension n and a maximum dimension n^2. Suppose we have r different diagonal entries, and there are k_i copies of diagonal entry λ_i. Each $k_i \geq 1$, and we have

$$k_1 + k_2 + \cdots + k_r = n.$$

Then by the block construction I mentioned above, the dimension of the space of matrices that commute with Λ is

$$k_1^2 + k_2^2 + \cdots + k_r^2.$$

The minimum is when $r = n$, so all $k_i = 1$, and the dimension is n

The maximum is when $r = 1$, and $k_1 = n$, the matrix is a scalar multiple of the identity matrix, and the dimension is n^2.