$$\phi = axy^2 + byz + cz^2x^2$$

$$\Rightarrow \nabla\phi = (ay^2 + 2cz^2x)\hat{i} + (yz + 2axy)\hat{j} + (by + 2cx^2)\hat{k}$$

$$\Rightarrow (\nabla\phi)_{(1,2,-1)} = (4a + 2c)\hat{i} + (4a - b)\hat{j} + (2b - 2c)\hat{k}.$$

in the direction parallel to z-axis

$$\Rightarrow 4a + 2c = 0$$

$$4a - b = 0 \Rightarrow 4a = b$$

$$\Rightarrow b + 2c = 0 \quad - (1)$$

$$|\nabla\phi| = \nabla\phi \cdot \hat{k}$$

$$= 2b - 2c = 64$$

$$b - c = 32 \quad - (2)$$

Solve (1) & (2)

$$b + 2c = 0$$

$$b - c = 32$$

$$3c = -32$$

$$\boxed{c = -\frac{32}{3}}$$

$$b = 32 - \frac{32}{3} = \boxed{\frac{64}{3} = b}$$

$$a = \frac{b}{4} = \frac{64}{3 \times 4} = \boxed{\frac{16}{3} = a}$$

$a = \frac{16}{3}$

$b = \frac{64}{3}$

$c = -\frac{32}{3}$