

we know,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$f(x) = e^{x^2} = 1 + x^2 + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \dots + \frac{(x^2)^{49}}{49!} + \frac{(x^2)^{50}}{50!} + \frac{(x^2)^{51}}{51!} + \dots$$

$$= 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots + \frac{x^{98}}{49!} + \frac{x^{100}}{50!} + \frac{x^{102}}{51!} + \dots$$

$$f^{100}(x) = 0 + 0 + 0 + \dots + 0 + 0 + \frac{100!}{50!} + \frac{x^{102}}{51!} + \dots$$

$$f^{100}(0) = 0 + 0 + \dots + 0 + \frac{100!}{50!} + 0 + 0 + \dots$$

Thus

$$f^{100}(0) = \frac{100!}{50!} \quad \rightarrow$$