$$\sum \frac{1}{n^2\left[1+\frac{1}{2}\sin\left(n\frac{\pi}{4}\right)\right]}$$

$$\sin\frac{n\pi}{4} = \left\{\frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}} \ldots\right.$$

$$-1 \le \sin\frac{n\pi}{4} \le 1$$

So,

$$\sum \frac{1}{n^2\left(1+\frac{n}{2}\right)} \le \sum \frac{1}{n^2\left[1+\frac{1}{2}\sin\left(n\frac{\pi}{4}\right)\right]}$$

$$\le \sum \frac{1}{n^2\left[1-\frac{n}{2}\right]}$$

lower bound $\rightarrow$ $\sum \frac{2}{n^2(2+n)}$

Upper bound $\rightarrow$ $\sum \frac{2}{n^2(2-n)}$