Newton Divided Differences and Nested Multiplication

The following algorithm overwrites given initial values $c_0 = y_0, \ldots, c_n = y_n$ with their Newton divided differences at distinct $x_0, \ldots, x_n$. The final $c_0, \ldots, c_n$ are the coefficients of the interpolating polynomial $p_n(x)$ in Newton form, i.e.,

$$p_n(x) = c_0 + c_1(x - x_)) + \ldots + c_n(x - x_0) \ldots (x - x_n).$$

<u>Newton Divided Differences:</u>

Given initial $c_0 = y_0, \ldots, c_n = y_n$ and distinct $x_0, \ldots, x_n$,

For $k = 1, \ldots, n$

    For $j = n, \ldots, k$

        Update $c_j \leftarrow (c_j - c_{j-1})/(x_j - x_{j-k})$.

The following algorithm evaluates the interpolating polynomial in Newton form at a point x, given the Newton divided differences $c_0, \ldots, c_n$ at distinct $x_0, \ldots, x_n$.

<u>Nested Multiplication:</u>

Given the coefficients $c_0, \ldots, c_n$ and distinct $x_0, \ldots, x_n$,

Set $pval = c_n$.

For $k = n - 1, \ldots, 0$

    Update $pval \leftarrow c_k + (x - x_k) \cdot pval$.