5 Previous Year Papers with Answer Key (2016 to 2020)

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Mathematical Statistics(MS)

Previous Year Solved Paper 2020

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ),** wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 – Q.10 carry one mark each.

1. Let E and F be two events. Then which one of the following statements is **NOT** ALWAYS TRUE?

(A) $P(E \cap F) \leq \max\{1 - P(E^c) - P(F^c), 0\}$

(B) $P(E \cup F) \geq \max\{P(E), P(F)\}$

(C) $P(E \cup F) \leq \min\{P(E), P(F), 1\}$

(D) $P(E \cap F) \leq \min\{P(E), P(F)\}$

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables having $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$. Define

$$W = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - X_j)^2$$

Then W, as an estimator of σ^2, is

(A) biased and consistent (B) unbiased and consistent

(C) biased and inconsistent (D) unbiased and inconsistent

3. The mean and the standard deviation of weights of ponies in a large animal shelter are 20 kg and 3 kg, respectively. A pony is selected at random from the shelter. Using Chebyshev's inequality, the value of the lower bound of the probability that the weight of the selected pony is between 14 kg and 26 kg is

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$

(C) 0 (D) 1

4. Let $X_1, X_2, \ldots, X_{10}$ be a random sample from N(1, 2) distribution. If

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i \text{ and } S^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2,$$

then $\text{Var}(S^2)$ equals

(A) $\frac{2}{5}$ (B) $\frac{4}{9}$

(C) $\frac{11}{9}$ (D) $\frac{8}{9}$

5. For real constants a and b, let

$$f(x) = \begin{cases} \dfrac{a\sin x - 2x}{x}, & x < 0 \\ bx, & x \geq 0 \end{cases}$$

If f is a differentiable function then the value of a + b is

(A) 0 (B) 1

(C) 2 (D) 3

6. The area of the region bounded by the curves $y_1(x) = x^4 - 2x^2$ and $y_2(x) = 2x^2$, $x \in \mathbb{R}$.

(A) $\frac{128}{15}$ (B) $\frac{129}{15}$

(C) $\frac{133}{15}$ (D) $\frac{134}{15}$

7. IF $\{x_n\}_{n \geq 1}$ is a sequence of real number such that $\lim_{n\to\infty} \frac{x_n}{n} = 0.001$, then

(A) $\{x_n\}_{n\geq1}$ is a bounded sequence

(B) $\{x_n\}_{n\geq1}$ is an unbounded sequence

(C) $\{x_n\}_{n\geq1}$ is a convergent sequence

(D) $\{x_n\}_{n\geq1}$ is a monotonically decreasing sequence

8. Let $\{x_n\}_{n \geq 1}$ be a sequence of i.i.d. random variables such that $E(X_i) = 1$ and $Var(X_i) = 1$, i = 1, 2, ... Then the approximate distribution of $\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_{2i} - X_{2i-1})$, for large n, is

(A) N(0, 1) (B) N(0, 2)

(C) N(0, 0.5) (D) N(0, 0.25)

9. Consider the following system of linear equations

$ax + 2y + z = 0$

$y + 5z = 0$

$by - 5z = -1$

Which one of the following statements is TRUE?

(A) The system has unique solution for a = 1, b = –1

(B) The system has unique solution for a = –1, b = 1

(C) The system has no solution for a = 1, b = 0

(D) The system has infinitely many solution for a = 0, b = 0

10. Let X be a random variable having Poisson (2) distribution. Then $E\left(\frac{1}{1+x}\right)$ equals

(A) $1 - e^{-2}$ (B) e^{-2}

(C) $\frac{1}{2}(1-e^{-2})$ (D) $\frac{1}{2}e^{-1}$

11. Which one of the following series is convergent?

(A) $\sum_{n=1}^{\infty}\left(\frac{5n+1}{4n+1}\right)^n$ (B) $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^n$

(C) $\sum_{n=1}^{\infty}\frac{\sin\left(\frac{1}{n}\right)}{n^{1/n}}$ (D) $\sum_{n=1}^{\infty}\sqrt{n}\left(1-\cos\left(\frac{1}{n}\right)\right)$

12. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation. If $T(1, 1, 0) = (2, 0, 0, 0)$, $T(1, 0, 1) = (2, 4, 0, 0)$ and $T(0, 1, 1) = (0, 0, 2, 0)$, then $T(1, 1, 1)$ equals

(A) $(1, 1, 1, 0)$ (B) $(0, 1, 1, 1)$

(C) $(2, 2, 1, 0)$ (D) $(0, 0, 0, 0)$

13. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability density function

$$f_\theta(x) = \begin{cases} \theta(1-x)^{\theta-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}, \quad \theta > 0$$

To test $H_0 : \theta = 1$ against $H_1 : \theta > 1$, the uniformly most powerful test of size $\alpha(0 < \alpha < 1)$ would reject H_0 if

(A) $-\sum_{i=1}^{n} \log_e(1-X_i)^2 < \chi^2_{2n,1-\alpha}$ (B) $-\sum_{i=1}^{n} \log_e(1-X_i)^2 < \chi^2_{n,1-\alpha}$

(C) $-\sum_{i=1}^{n} \log_e(1-X_i)^2 < \chi^2_{2n,\alpha}$ (D) $-\sum_{i=1}^{n} \log_e(1-X_i)^2 < \chi^2_{n,\alpha}$

14. Let X_1, X_2, X_3, X_4, X_5 be a random sample from $N(0, 1)$ distribution and let $W = \dfrac{X_1^2}{X_2^2 + X_3^2 + X_4^2 + X_5^2}$. Then $E(W)$ equals

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{5}$

15. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta - 0.5, \theta + 0.5)$ distribution, where $\theta \in \mathbb{R}$. If $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$ then which one of the following estimators is **NOT** a maximum likelihood estimator of θ?

(A) $\frac{1}{2}\left(X_{(1)} + X_{(n)}\right)$ (B) $\frac{1}{4}(3X_{(1)} + X_{(n)} + 1)$

(C) $\frac{1}{4}\left(X_{(1)} + 3X_{(n)} - 1\right)$ (D) $\frac{1}{2}(3X_{(n)} - X_{(1)} - 2)$

16. Let $X_1, X_2, \ldots, X_n$ be a random sample from $\text{Exp}(\theta)$ distribution, where $\theta \in (0, \infty)$. If $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then a 95% confidence interval for θ is

(A) $\left(0, \dfrac{\chi^2_{2n,0.95}}{n\bar{X}}\right]$ (B) $\left[\dfrac{\chi^2_{2n,0.95}}{n\bar{X}}, \infty\right)$

(C) $\left(0, \dfrac{\chi^2_{2n,0.05}}{2n\bar{X}}\right]$ (D) $\left[\dfrac{\chi^2_{2n,0.05}}{2n\bar{X}}, \infty\right)$

17. The value of the integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ is

(A) $\frac{17}{9}$ (B) $\frac{16}{9}$

(C) $\frac{14}{9}$ (D) $\frac{13}{9}$

18. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma^2)$ distribution and $Y_1, Y_2, \ldots, Y_m$ be random sample from $N(\mu_2, \sigma^2)$ distribution, where $\mu_i \in \mathbb{R}$, i = 1, 2 and $\sigma > 0$. Suppose that the two random samples are independent. Define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \text{ and } W = \frac{\sqrt{mn}\left(\bar{X}-\mu_1\right)}{\sqrt{\sum_{i=1}^{m}\left(Y_i-\mu_2\right)^2}}$$

Then which one of the following statements is TRUE for all positive integers m and n?

(A) $W \sim t_m$ (B) $W \sim t_n$

(C) $W^2 \sim F_{m,1}$ (D) $W^2 \sim F_{m,n}$

19. Let M be an n × n non-zero skew symmetric matrix. Then the matrix $(I_n - M)(I_n + M)^{-1}$ is always

(A) singular (B) symmetric

(C) orthogonal (D) idempotent

20. Let α and β be two real numbers. If $\lim_{x\to 0} \frac{\tan 2 - 2\sin \alpha x}{x(1-\cos 2x)} = \beta$ then $\alpha + \beta$ equals

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{3}{2}$ (D) $\frac{5}{2}$

21. Let the joint probability density function of (X, Y) be

$$f(x,y) = \begin{cases} 2e^{-(x+y)}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Then $P\left(X < \frac{y}{2}\right)$ equals

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

22. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 1$, $a_2 = 7$ and $a_{n+1} = \frac{a_n + a_{n-1}}{2}, n \geq 2$. Assuming that $\lim_{n\to\infty} a_n$ exists, the value of $\lim_{n\to\infty} a_n$ is

(A) $\frac{19}{4}$ (B) $\frac{9}{2}$

(C) 5 (D) $\frac{21}{4}$

23. Let $X_1, X_2, \ldots, X_n$ be a random sample from U(1, 2) distribution and let $Y_1, Y_2, \ldots, Y_n$ be a random sample from U(0, 1) distribution. Suppose that the two random samples are independent. Define

$$Z_i = \begin{cases} 1, & \text{if } X_i Y_i < 1 \\ 0, & \text{otherwise} \end{cases}, \ i = 1, 2, \ldots. n$$

If $\lim_{n\to\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n} Z_i - \theta\right| < \epsilon\right) = 1$, for all $\epsilon > 0$, then θ equal

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\log_e \frac{3}{2}$ (D) $\log_e 2$

24. If the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$ is given by

$$\int_0^{\alpha}\int_{\beta(y)}^{2y}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} dx\,dz\,dy$$

then

(A) $\alpha = 2$ and $\beta(y) = y$, $y \in [0, 2]$ (B) $\alpha = 1$ and $\beta(y) = y^2$, $y \in [0, 1]$

(C) $\alpha = 2$ and $\beta(y) = y^2$, $y \in [0, 2]$ (D) $\alpha = 1$ and $\beta(y) = y$, $y \in [0, 1]$

25. Consider the simple linear regression model

$$Y_i = \alpha + \beta x_i + \epsilon_i, \quad i = 1, 2, \ldots, n,$$

where ϵ_i's are i.i.d. random variable with mean 0 and variance $\sigma^2 \in (0, \infty)$. Suppose that we have a data set $(x_1, y_1), \ldots, (x_n, y_n)$ with $n = 20$, $\sum_{i=1}^{n} x_i = 100$, $\sum_{i=1}^{n} y_i = 50$, $\sum_{i=1}^{n} x_i^2 = 600$, $\sum_{i=1}^{n} y_i^2 = 500$ and $\sum_{i=1}^{n} x_i y_i = 400$. Then the least square estimates of α and β are, respectively.

(A) 5 and $\frac{3}{2}$ (B) -5 and $\frac{3}{2}$

(C) 5 and $-\frac{3}{2}$ (D) -5 and $-\frac{3}{2}$

26. Let Z_1 and Z_2 be i.i.d. N(0, 1) random variables. If $Y = Z_1^2 + Z_2^2$, then $P(Y > 4)$ equals

(A) e^{-2} (B) $1 - e^{-2}$

(C) $\frac{1}{2}e^{-2}$ (D) e^{-4}

27. Let f : $\mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{2x^3 + 3y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Let $f_x(0, 0)$ and $f_y(0, 0)$ denote the first order partial derivatives of f(x, y) with respect to x and y, respectively, at the point (0, 0). Then which one of the following statements is TRUE?

(A) f is continuous at (0, 0) but $f_x(0, 0)$ and $f_y(0, 0)$ do not exist

(B) f is differentiable at (0, 0)

(C) f is not differentiable at (0, 0)

(D) f is not continuous at (0, 0) but $f_x(0, 0)$ and $f_y(0, 0)$ exist

28. A packet contains 10 distinguishable firecrackers out of which 4 are defective. If three firecrackers are drawn at random (without replacement) from the packet, then the probability that all three firecrackers are defective equals

(A) $\frac{1}{10}$ (B) $\frac{1}{20}$

(C) $\frac{1}{30}$ (D) $\frac{1}{40}$

29. Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{3}$. Let X denote the number of trials required to get the second success. Then $P(X \geq 5)$ equals

(A) $\frac{3}{7}$ (B) $\frac{16}{27}$

(C) $\frac{16}{21}$ (D) $\frac{9}{13}$

30. Let X_1, X_2, X_3, X_4 be i.i.d. random variables having a continuous distribution. Then $P(X_3 < X_2 < \max(X_1, X_4))$ equals

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{6}$

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

Q.31 – Q.40 carry two marks each.

31. Let the sequence $\{x_n\}_{n\geq 1}$ be given by $x_n = \sin\frac{n\pi}{6}, n = 1, 2, ...$ Then which of the following statements is/are TRUE?

(A) The sequence $\{x_n\}_{n\geq 1}$ has a subsequence that converges to $\frac{1}{2}$

(B) $\lim\sup_{n\to\infty} x_n = 1$

(C) $\lim\inf_{n\to\infty} x_n = -1$

(D) The sequence $\{x_n\}_{n\geq 1}$ has a subsequence that converges to $\frac{1}{\sqrt{2}}$

32. Consider the following two probability density functions (pdfs)

$$f_0(x) = \begin{cases} 2x, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases} \text{ and } f_1(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Let X be a random variable having pdf $f \in p = \{f_0, f_1\}$. Consider testing $H_0 : f(x) = f_0(x), \forall x \in [0, 1]$ against $H_1: f(x) = f_1(x), \forall x \in [0, 1]$ at $\alpha = 0.05$ level of significance. For which of the following observed values of random observation X, the most powerful test would reject H_0?

(A) 0.19

(B) 0.22

(C) 0.25

(D) 0.28

33. The arc length of the parabola $y^2 = 2x$ intercepted between the points of intersection of the parabola $y^2 = 2x$ and the straight line $y = 2x$ equals

(A) $\int_0^1 \sqrt{1+y^2}\, dy$

(B) $\int_0^1 \sqrt{1+4y^2}\, dy$

(C) $\int_0^{1/2} \frac{\sqrt{1+2x}}{\sqrt{2x}} dx$

(D) $\int_0^{1/2} \frac{\sqrt{1+4x}}{\sqrt{2x}} dx$

34. Let (X, Y) have the joint probability mass function

$$f(x,y) = \begin{cases} \binom{x+1}{y}\binom{16}{x}\left(\frac{1}{6}\right)^y\left(\frac{5}{6}\right)^{x+1-y}\left(\frac{1}{2}\right)^{16}, & y = 0,1,...,x+1;\ x = 0,1,...,16 \\ 0, & \text{otherwise} \end{cases}$$

Then which of the following statements is/are TRUE?

(A) $E(Y) = \frac{3}{2}$

(B) $Var(Y) = \frac{49}{36}$

(C) $E(XY) = \frac{37}{3}$

(D) $Var(X) = 3$

35. Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{5}$. Then which of the following statements is/are TRUE?

(A) Expected number of trials required to get the first success is 5.

(B) Expected number of successes in first 50 trials is 10

(C) Expected number of failures preceding the first success is 4

(D) Expected number of trials required to get the second success is 10

36. Let X_1, X_2, X_3 be i.i.d. N(0, 1) random variables. Then which of the following statements is/are TRUE?

(A) $\frac{\sqrt{2}x_1}{\sqrt{x_2^2 + x_3^2}} \sim t_2$

(B) $\frac{\sqrt{2}x_1}{|x_2 + x_3|} \sim t_1$

(C) $\frac{(x_1 - x_2)^2}{(x_1 + x_2)^2} \sim F_{1,1}$

(D) $\sum_{i=1}^{3} X_i^2 \sim \chi_2^2$

37. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables such that

$$P(X_1 = 0) = \frac{1}{4} = 1 - P(X_1 = 1)$$

Define

$$U_n = \frac{1}{n}\sum_{i=1}^{n} X_i \text{ and } V_n = \frac{1}{n}\sum_{i=1}^{n}(1 - X_i)^2,\ n = 1, 2, \ldots$$

Then which of the following statements is/are TRUE?

(A) $\lim_{n\to\infty} P\left(\left|U_n - \frac{3}{4}\right| < \frac{1}{100}\right) = 1$

(B) $\lim_{n\to\infty} P\left(\left|U_n - \frac{3}{4}\right| > \frac{1}{100}\right) = 0$

(C) $\lim_{n\to\infty} P\left(\sqrt{n}\left(U_n - \frac{3}{4}\right) \leq 1\right) = \phi(2)$

(D) $\lim_{n\to\infty} P\left(\sqrt{n}\left(V_n - \frac{1}{4}\right) \leq 1\right) = \phi\left(\frac{4}{\sqrt{3}}\right)$

38. For real constants a and b, let

$$M = \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ a & b \end{vmatrix}$$

be an orthogonal matrix. Then which of the following statements is/are always TRUE?

(A) a + b = 0

(B) $b = \sqrt{1 - a^2}$

(C) $ab = -\frac{1}{2}$

(D) $M^2 = I_2$

39. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^2(2 - y) - y^3 + 3y^2 + 9y$, where $(x, y) \in \mathbb{R}^2$. Which of the following is/are saddle point(s) of f?

(A) (0, – 1)

(B) (0, 3)

(C) (3, 2)

(D) (–3, 2)

40. Let $X_1, X_2, \ldots, X_n$ be i.i.d. Poisson (λ) random variables, where $\lambda > 0$. Define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \text{ and } S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$$

Then which of the following statements is/are TURE?

(A) $\text{Var}(\bar{X}) < \text{Var}(S^2)$

(B) $\text{Var}(\bar{X}) = \text{Var}(S^2)$

(C) $\text{Var}(\bar{X})$ attains the Cramer-Rao lower bound

(D) $E(\bar{X}) = E(S^2)$

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

Q.41 – Q.50 carry one mark each.

41. The value of the integral $\int_0^1 \int_{y^2}^1 \frac{e^x}{\sqrt{x}}\,dx\,dy$ equals ______ *(round off to two decimal places)*

42. $\lim\limits_{n\to\infty} \frac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{n^6+1}}$ equals ______.

43. Let the sample mean based on a random sample from Exp(λ) distribution be 3.7. Then the maximum likelihood estimate of $1 - e^{-\lambda}$ equals ______ *(round off to two decimal places)*

44. The maximum value of the function $y = \frac{x^2}{x^4+4}, x \in \mathbb{R}$, is ______.

45. Let X be a random variable having the Poisson(4) distribution and let E be an event such that $P(E|X = i) = 1 - 2^{-i}$, i = 0, 1, 2,... Then P(E) equals ______ *(round off to two decimal places)*

46. Let $U \sim F_{5,8}$ and $V \sim F_{8,5}$. If $P[U > 3.69] = 0.05$, then the value of c such that $P[V > c] = 0.95$ equals ______ *(round off to two decimal places)*

47. Let X_1, X_2 and X_3 be independent random variables such that $X_1 \sim N(47, 10)$, $X_2 \sim N(55, 15)$ and $X_3 \sim N(60, 14)$. Then $P(X_1 + X_2 \geq 2X_3)$ equals ______ *(round off to two decimal places)*

48. Let (X, Y) have the joint probability density function

$$f(x,y) = \begin{cases} \frac{3}{4}(y-x), & 0 < x < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Then the conditional expectation $E(X|Y = 1)$ equals ______ *(round off to two decimal places)*

49. The rank of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 5 & 6 & 4 \\ 2 & 6 & 8 & 5 \end{bmatrix}$$

is ________.

50. Let X be a single observation drawn from U(0, θ) distribution, where $\theta \in [1, 2]$. To test $H_0 : \theta = 1$ against $H_1 : \theta = 2$ consider the test procedure that rejects H_0 if and only if $X > 0.75$. If the probabilities of Type-I and Type-II errors are α and β, respectively, then $\alpha + \beta$ equals ________ *(round off to two decimal places)*

51. Let $f : [-1,3] \to \mathbb{R}$ be a continuous function such that f is differentiable on (–1, 3), $|f'(x)| \le \frac{3}{2}, \forall x \in (-1,3)$, f(–1) = 1 and f(3) = 7. Then f(1) equals ________.

52. Let P be a 3 × 3 matrix having characteristic roots $\lambda_1 = \frac{2}{3}$, $\lambda_2 = 0$ and $\lambda_3 = 1$. Define $Q = 3P^3 - P^2 - P + I_3$ and $R = 3P^3 - 2P$. If $\alpha = \det(Q)$ and $\beta = \text{trace}(R)$, then $\alpha + \beta$ equals ________ *(round off to two decimal places).*

53. Let $x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 1, x_5 = 0, x_6 = 1$ be the data on a random sample of size 6 from Bin(1, θ) distribution, where $\theta \in (0, 1)$. Then the uniformly minimum variance unbiased estimate of $\theta(1 + \theta)$ equals ________.

54. Let α be the real number such that the coefficient of x^{125} in Maclaurin's series of $(x+\alpha^3)e^x$ is $\frac{28}{124!}$. Then α equals ________.

55. Let $x_1 = 1, x_2 = 4$ be the data on a random sample of size 2 from a Poisson(θ) distribution, where $\theta \in (0, \infty)$. Let $\hat{\psi}$ be the uniformly minimum variance unbiased estimate of $\Psi(\theta) = \sum_{k=4}^{\infty} \frac{e^{-\theta}\theta^k}{k!}$ based on the given data. The $\hat{\psi}$ equals ________ *(round off to two decimal places)*

56. Let X and Y be independent random variables with respective moment generating functions

$$M_x(t) = \frac{(8+e^t)^2}{81} \text{ and } M_y(t) = \frac{(1+3e^t)^3}{64}, -\infty < t < \infty$$

Then P(X + Y = 1) equals ________ *(round off to two decimal places)*

57. Let X be a random variable having N(θ, 1) distribution, where $\theta \in \mathbb{R}$. Consider testing $H_0 : \theta = 0$ against $H_1 : \theta \neq 0$ at $\alpha = 0.617$ level of significance. The power of the likelihood ration test at $\theta = 1$ equals _______ *(round off to two decimal places)*

58. Consider the matrix

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

Let P be a non-singular matrix such that $P^{-1}MP$ is a diagonal matrix. Then the trace of the matrix $P^{-1}M^3P$ equals _______ .

59. Let X be a random variable having U(0, 10) distribution and Y = X – [X], where [X] denotes the greatest integer less than or equal to X. Then P(Y > 0.25) equals _______ .

60. A computer lab has two printers handling certain types of printing jobs. Printer-I and Printer-II handle 40% and 60% of the jobs, respectively. For a typical printing job, printing time (in minutes) of printer-I follows N(10, 4) distribution and that of Printer-II follows U(1, 21) distribution. If a randomly selected printing job is found to have been completed in less than 10 minutes, then the conditional probability that it was handled by the Printer-II equals ________ *(round off to two decimal places)*

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
A	A	A	D	C	A	B	B	B	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	C	A	A	D	C	B	A	C	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	C	D	C	B	A	C	C	B	C

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
A,B,C	A,B	A,C	A,B	A,B,C,D	A,B,C	A,B,D	A,C	C,D	A,C,D

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	42	43	44	45	46	47	48	49	50
1.70-1.74	4	0.22-0.26	0.25	0.84-0.88	0.25-0.29	0.01-0.03	0.32-0.34	3	0.61-0.65
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
4	2.00-2.30	0.70	15	0.17-0.20	0.10-0.12	0.74-0.77	134	0.75	0.55-0.60

Mathematical Statistics (MS)

Previous Year Solved Paper 2019

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ),** wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.

Special Instructions/Useful Data

$\mathbb{R}$	The set of all real numbers
P^T	Transpose of the matrix P
$\mathbb{R}^n$	$\left\{\begin{bmatrix} x_1 \\ : \\ x_n \end{bmatrix} : x_i \in \mathbb{R},\ i = 1, 2, \ldots, n\right\}$
f'	Derivative of the differentiable function f
I_n	$n \times n$ identity matrix
P(E)	Probability of the event E
E(X)	Expectation of the random variable X
Var(X)	Variance of the random variable X
i.i.d.	Independently and identically distributed
U(a, b)	Continuous uniform distribution on (a, b), $-\infty < a < b < \infty$
Exp (λ)	Exponential distribution with probability density function, for $\lambda > 0$,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}$$

$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
$\Phi(a)$	$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-\frac{u^2}{2}} du$
χ_n^2	Central Chi-squared distribution with n degrees of freedom
$t_{n,\alpha}$	A constant such that $P(X > t_{n,\alpha}) = \alpha$, where X has Student's t-distribution with n degrees of freedom
n!	$n(n-1)\ldots\ldots 3.\ 2.\ 1$ for $n = 1, 2, 3\ldots\ldots$, and $0! = 1$

$$\Phi(1.65) = 0.950, \quad \Phi(1.96) = 0.975$$

$$t_{4,\ 0.05} = 2.132, \quad t_{4,\ 0.10} = 1.533$$

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 – Q.10 carry one mark each.

1. Let $\{X_n\}_{n \geq 1}$ be a sequence of positive real numbers. Which one of the following statements is always TRUE ?

(A) If $\{x_n\}_{n \geq 1}$ is a convergent sequence, then $\{x_n\}_{n \geq 1}$ is monotone

(B) If $\{x_n^2\}_{n \geq 1}$ is a convergent sequence, then the sequence $\{x_n\}_{n \geq 1}$ does not converge

(C) If the sequence $\{|x_{n+1} - x_n|\}_{n \geq 1}$ converges to 0, then the series $\sum_{n=1}^{\infty} x_n$ is convergent

(D) If $\{x_n\}_{n \geq 1}$ is a convergent sequence, then $\{e^{x_n}\}_{n \geq 1}$ is also a convergent sequence

2. Consider the function $f(x, y) = x^3 - 3xy^2$, $x, y \in R$. Which one of the following statement is TRUE ?

(A) f has a local minimum at (0, 0)
(B) f has a local maximum at (0, 0)
(C) f has global maximum at (0, 0)
(D) f has a saddle point at (0, 0)

3. If $F(x) = \int_{x^3}^{4} \sqrt{4+t^2}\,dt$, for $x \in \mathbb{R}$, then F'(1) equals

(A) $-3\sqrt{5}$
(B) $-2\sqrt{5}$
(C) $2\sqrt{5}$
(D) $3\sqrt{5}$

4. Let $T : R^2 \to R^2$ be a linear transformation such that $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right)=\begin{bmatrix}1\\0\end{bmatrix}$ and $T\left(\begin{bmatrix}2\\1\end{bmatrix}\right)=\begin{bmatrix}0\\1\end{bmatrix}$

Suppose that $\begin{bmatrix}3\\-2\end{bmatrix}=\alpha\begin{bmatrix}1\\2\end{bmatrix}+\beta\begin{bmatrix}2\\1\end{bmatrix}$ and $T\left(\begin{bmatrix}3\\-2\end{bmatrix}\right)=\begin{bmatrix}a\\b\end{bmatrix}$. Then $\alpha + \beta + a + b$ equals

(A) $\frac{2}{3}$
(B) $\frac{4}{3}$
(C) $\frac{5}{3}$
(D) $\frac{7}{3}$

5. Two biased coins C_1 and C_2 have probabilities $\frac{2}{3}$ and $\frac{3}{4}$, respectively, when tossed. If both coins are tossed independently two times each, then the probability of getting exactly two heads out of these four tosses is

(A) $\frac{1}{4}$
(B) $\frac{37}{144}$
(C) $\frac{41}{144}$
(D) $\frac{49}{144}$

6. Let X be a discrete random variable with the probability mass function

$$P(X = n) = \begin{cases} \frac{-2c}{n} & , \quad n = -1, -2 \\ d & , \quad n = 0 \\ cn & , \quad n = 1, 2 \\ 0 & , \quad \text{otherwise,} \end{cases}$$

where c and d are positive real numbers. If $P(|X| \leq 1) = 3/4$, then E(X) equals

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

7. Let X be a Poisson random variable and $P(X = 1) + 2\,P(X = 0) = 12\,P(X = 2)$. Which one of the following statements is TRUE ?

(A) $0.40 < P(X = 0) \leq 0.45$ (B) $0.45 < P(X = 0) \leq 0.50$

(C) $0.50 < P(X = 0) \leq 0.55$ (D) $0.55 < P(X = 0) \leq 0.60$

8. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. discrete random variables with the probability mass function

$$P(X_1 = m) = \begin{cases} \frac{(\log_e 2)^m}{2(m!)}, & m = 0, 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

If $S_n = X_1 + X_2 + \ldots + X_n$, then which one of the following sequences of random variables converges to 0 in probability ?

(A) $\frac{S_n}{n \log_e 2}$ (B) $\frac{S_n - n\log_e 2}{n}$

(C) $\frac{S_n - \log_e 2}{n}$ (D) $\frac{S_n - n}{\log_e 2}$

9. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function

$$f(x) = \frac{1}{2\sqrt{2\pi}}\left[e^{-\frac{1}{2}(x-2\mu)^2} + e^{-\frac{1}{2}(x-4\mu)^2}\right], -\infty < x < \infty.$$

If $T = X_1 + X_2 + \ldots + X_n$, then which one of the following is an unbiased estimator of μ ?

(A) $\frac{T}{n}$ (B) $\frac{T}{2n}$

(C) $\frac{T}{3n}$ (D) $\frac{T}{4n}$

10. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. Instead of observing $X_1, X_2, \ldots, X_n$, we observe $Y_1, Y_2, \ldots, Y_n$, where $Y_i = e^{X_i}$, $i = 1, 2, \ldots, n$. To test the hypothesis

$H_0: \theta = 1$ against $H_1: \theta \neq 1$

based on the random sample $Y_1, Y_2, \ldots, Y_n$, the rejection region of the likelihood ratio test is of the form, for some $c_1 < c_2$.

(A) $\sum_{i=1}^{n} Y_i \le c_1$ or $\sum_{i=1}^{n} Y_i \ge c_2$ (B) $c_1 \le \sum_{i=1}^{n} Y_i \le c_2$

(C) $c_1 \le \sum_{i=1}^{n} \log_e Y_i \le c_2$ (D) $\sum_{i=1}^{n} \log_e Y_i \le c_1$ or $\sum_{i=1}^{n} \log_e Y_i \ge c_2$

Q. 11 – Q. 30 carry two marks each.

11. $\sum_{n=4}^{\infty} \frac{6}{n^2 - 4n + 3}$ equals.

(A) $\frac{5}{2}$ (B) 3

(C) $\frac{7}{2}$ (D) $\frac{9}{2}$

12. $\lim_{n\to\infty} \frac{1 + \frac{1}{2} + \ldots + \frac{1}{n}}{\left(\pi^n + e^n\right)^{1/n} \log_e n}$ equals

(A) $\frac{1}{\pi}$ (B) $\frac{1}{e}$

(C) $\frac{e}{\pi}$ (D) $\frac{\pi}{e}$

13. A possible value of $b \in \mathbb{R}$ for which the equation $x^4 + bx^3 + 1 = 0$ has no real root is

(A) $\frac{-11}{5}$ (B) $\frac{-3}{2}$

(C) 2 (D) $\frac{5}{2}$

14. Let the Taylor polynomial of degree 20 for $\frac{1}{(1-x)^3}$ at $x = 0$ be $\sum_{n=0}^{20} a_n x^n$. Then a_{15} is :

(A) 136 (B) 120

(C) 60 (D) 272

15. The length of the curve $y = \frac{3}{4}x^{4/3} - \frac{3}{8}x^{2/3} + 7$ from $x = 1$ to $x = 8$ equals

(A) $\frac{99}{8}$ (B) $\frac{117}{8}$

(C) $\frac{99}{4}$ (D) $\frac{117}{4}$

16. The volume of the solid generated by revolving the region bounded by the parabola $x = 2y^2 + 4$ and the line $x = 6$ about the line $x = 6$ is :

(A) $\frac{78\pi}{15}$ (B) $\frac{91\pi}{15}$

(C) $\frac{64\pi}{15}$ (D) $\frac{117\pi}{15}$

17. Let P be a 3 × 3 non-null real matrix. If there exist a 3 × 2 real matrix Q and a 2 × 3 real matrix R such that P = QR, then

(A) $Px = 0$ has a unique solution, where $0 \in R^3$

(B) there exists $b \in R^3$ such that $Px = b$ has no solution

(C) there exists a non-zero $b \in R^3$ such that $Px = b$ has a unique solution

(D) there exists a non-zero $b \in R^3$ such that $P^T x = b$ has a unique solution

18. If $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & -1 \end{bmatrix}$ and $6P^{-1} = aI_3 + bP - P^2$, then the ordered pair (a, b) is

(A) (3, 2) (B) (2, 3)

(C) (4, 5) (D) (5, 4)

19. Let E, F and G be any three events with P(E) = 0.3, P(F/E) = 0.2, P(G/E) = 0.1 and $P(F \cap G/E) = 0.05$. Then $P(E - (F \cup G))$ equals

(A) 0.155 (B) 0.175

(C) 0.225 (D) 0.255

20. Let E and F be any two independent events with $0 < P(E) < 1$ and $0 < P(F) < 1$. Which one of the following statements is NOT TRUE ?

(A) P (Neither E nor F occurs) = (P(E) – 1) (P(F) – 1)

(B) P (Exactly one of E and F occurs) = P(E) + P(F) – P(E)P(F)

(C) P(E occurs but F does not occur) = $P(E) - P(E \cap F)$

(D) P(E occurs given that F does not occur) = P(E)

21. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{1}{3} x^7 e^{-x^2}, & x > 0, \\ 0, & \text{otherwise} \end{cases}.$$

Then the distribution of the random variable $W = 2X^2$ is-

(A) χ_2^2 (B) χ_4^2

(C) χ_6^2 (D) χ_8^2

22. Let X be a continuous random variable with the probability density function

$$f(x) = \frac{e^x}{\left(1+e^x\right)^2}, -\infty < x < \infty.$$

The E(X) and P(X > 1), respectively, are

(A) 1 and $(1 + e)^{-1}$ (B) 0 and $2(1 + e)^{-2}$

(C) 2 and $(2 + 2e)^{-1}$ (D) 0 and $(1 + e)^{-1}$

23. The lifetime (in years) of bulbs is distributed as an Exp(1) random variable. Using Poisson approximation to the binomial distribution, the probability (round off to 2 decimal places) that out of the fifty randomly chosen bulbs at most one fails within one month equals

(A) 0.05 (B) 0.07

(C) 0.09 (D) 0.11

24. Let X follow a beta distribution with parameters m (> 0) and 2. If $P\left(x \le \frac{1}{2}\right) = \frac{1}{2}$, then Var (X) equals :

(A) $\frac{1}{10}$ (B) $\frac{1}{20}$

(C) $\frac{1}{25}$ (D) $\frac{1}{40}$

25. Let X_1, X_2 and X_3 be i.i.d. U(0, 1) random variables. Then $P(X_1 > X_2 + X_3)$ equals-

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

26. Let X and Y be i.i.d. U(0, 1) random variables. Then E(X|X > Y) equals

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

27. Let –1 and 1 be the observed values of a random sample of size two from $N(\theta, \theta)$ distribution. The maximum likelihood estimate of θ is-

(A) 0 (B) 2

(C) $\frac{-\sqrt{5}-1}{2}$ (D) $\frac{\sqrt{5}-1}{2}$

28. Let X_1 and X_2 be a random sample from a continuous distribution with the probability density function

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x-\theta}{\theta}}, & x > \theta \\ 0, & \text{otherwise} \end{cases},$$

where $\theta > 0$. If $X_{(1)} = \min\{X_1, X_2\}$ and $\bar{X} = \frac{(X_1 + X_2)}{2}$, then which one of the following statement is TRUE ?

(A) $(\bar{X}, X_{(1)})$ is sufficient and complete

(B) $(\bar{X}, X_{(1)})$ is sufficient but not complete

(C) $(\bar{X}, X_{(1)})$ is complete but not sufficient

(D) $(\bar{X}, X_{(1)})$ is neither sufficient nor complete

29. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function $f(x)$. To test the hypothesis

H_0: $f(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}, -\infty < x < \infty$ against H_1: $f(x) = e^{-2|x|}$, $-\infty < x < \infty$, the rejection region of the most powerful size α test is of the form, for some $c > 0$,

(A) $\sum_{i=1}^{n} (X_i - 1)^2 \geq c$

(B) $\sum_{i=1}^{n} (X_i - 1)^2 \leq c$

(C) $\sum_{i=1}^{n} (|X_i| - 1)^2 \geq c$

(D) $\sum_{i=1}^{n} (|X_i| - 1)^2 \leq c$

30. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. To test H_0: $\theta = 0$ against H_1: $\theta = 1$, assume that the critical region is given by $\frac{1}{n}\sum_{i=1}^{n} X_i > \frac{3}{4}$. Then the minimum sample size required so that P(Type 1 error) ≤ 0.05 is :

(A) 3

(B) 4

(C) 5

(D) 6

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ

Q.31-Q.40 carry two marks each.

31. Let $\{x_n\}_{n \geq 1}$ be a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} x_n$ converges. Which of the following statements is (are) always TRUE ?

(A) The series $\sum_{n=1}^{\infty} \sqrt{x_n x_{n+1}}$ converges

(B) $\lim_{n\to\infty} n x_n = 0$

(C) The series $\sum_{n=1}^{\infty} \sin^2 x_n$ converges

(D) The series $\sum_{n=1}^{\infty} \frac{\sqrt{x_n}}{1+\sqrt{x_n}}$ converges

32. Let f : R $\to$ R be continuous on R and differentiable on $(-\infty, 0) \cup (0, \infty)$. Which of the following statements is (are) always TRUE ?

(A) If f is differentiable at 0 and f'(0) = 0, then f has a local maximum or a local minimum at 0

(B) If f has a local minimum at 0, then f is differentiable at 0 and f '(0) = 0

(C) If f'(x) < 0 for all x < 0 and f'(x) > 0 for all x > 0, then f has a global maximum at 0

(D) If f'(x) > 0 for all x < 0 and f'(x) < 0 for all x > 0, then f has a global maximum at 0

33. Let P be a 2 × 2 real matrix such that every non-zero vector in R^2 is an eigenvector of P. Suppose that λ_1 and λ_2 denote the eigenvalues of P and $P\begin{bmatrix}\sqrt{2}\\ \sqrt{3}\end{bmatrix} = \begin{bmatrix}2\\ t\end{bmatrix}$ for some $t \in R$. Which of the following statements is (are) TRUE ?

(A) $\lambda_1 \neq \lambda_2$

(B) $\lambda_1\lambda_2 = 2$

(C) $\sqrt{2}$ is an eigenvalue of P

(D) $\sqrt{3}$ is an eigenvalue of P

34. Let P be an n × n non-null real skew-symmetric matrix, where n is even. Which of the following statements is (are) always TRUE ?

(A) Px = 0 has infinitely many solutions, where $0 \in R^n$

(B) $Px = \lambda x$ has a unique solution for every non-zero $\lambda \in R$

(C) If $Q = (I_n + P)(I_n - P)^{-1}$, then $Q^TQ = I_n$

(D) The sum of all the eigenvalues of P is zero

35. Let X be a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ \dfrac{1+x^2}{10} & , \quad 0 \le x < 1, \\ \dfrac{3+x^2}{10} & , \quad 1 \le x < 2 \\ 1 & , \quad x \ge 2 \end{cases}$$

Which of the following statements is (are) TRUE ?

(A) $P(1 < X < 2) = \dfrac{3}{10}$

(B) $P(1 < X \le 2) = \dfrac{3}{5}$

(C) $P(1 \le X < 2) = \dfrac{1}{2}$

(D) $P(1 \le X \le 2) = \dfrac{4}{5}$

36. Let X and Y be i.i.d. Exp (λ) random variables. If Z = max {X – Y, 0}, then which of the following statements is (are) TRUE ?

(A) $P(Z = 0) = \dfrac{1}{2}$

(B) the cumulative distribution function of Z is $F(z) = \begin{cases} 0, & z < 0 \\ 1 - \dfrac{1}{2}e^{-\lambda z}, & z \ge 0 \end{cases}$

(C) $P(Z = 0) = 0$

(D) The cumulative distribution function of Z is $F(z) = \begin{cases} 0, & z < 0 \\ 1 - e^{-\lambda z/2}, & z \ge 0 \end{cases}$

37. Let the discrete random variable X and Y have the joint probability mass function

$$P(X = m, Y = n) = \begin{cases} \dfrac{e^{-2}}{m!n!}, & m = 0,1,2,....; n = 0,1,2,...., \\ 0, & \text{otherwise.} \end{cases}$$

Which of the following statements is (are) TRUE ?

(A) The marginal distribution of X is Poisson with mean 2

(B) The random variables X and Y are independent

(C) The covariance between X and X + $\sqrt{3}Y$ is 1.

(D) P(Y = n) = (n + 1) P(Y = n + 1) for n = 0, 1, 2,.....

38. Let $X_1, X_2, ...$, be a sequence of i.i.d. continuous random variables with the probability density function

$$f(x) = \begin{cases} 2e^{-2\left(x-\frac{1}{2}\right)}, & x \geq \dfrac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

If $S_n = X_1 + X_2 + ...+X_n$ and $\bar{X}_n = S_n / n$, then the distributions of which of the following sequences of random variables converge(s) to a normal distribution with mean 0 and a finite variance ?

(A) $\dfrac{S_n - n}{\sqrt{n}}$

(B) $\dfrac{S_n}{\sqrt{n}}$

(C) $\sqrt{n}\left(\bar{X}_n - 1\right)$

(D) $\dfrac{\sqrt{n}\left(\bar{X}_n - 1\right)}{2}$

39. Let $X_1, X_2, ..., X_n$ be a random sample from a $U(\theta, 0)$ distribution, where $\theta < 0$. If $T_n = \min\{X_1, X_2, ..., X_n\}$, then which of the following sequences of estimators is (are) consistent for θ ?

(A) T_n

(B) $T_n - 1$

(C) $T_n + \dfrac{1}{n}$

(D) $T_n - 1 - \dfrac{1}{n^2}$

40. Let $X_1, X_2, ...,X_n$ be a random sample from a continuous distribution with the probability density function, for $\lambda > 0$,

$$f(x) = \begin{cases} 2\lambda x e^{-\lambda x^2}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

To test the hypothesis $H_0: \lambda = \dfrac{1}{2}$ against $H_1: \lambda = \dfrac{3}{4}$ at the level α $(0 < \alpha < 1)$, which of the following statements is (are) TRUE ?

(A) The most powerful test exists for each value of α

(B) The most powerful test does not exist for some values of α

(C) If the most powerful test exists, it is of the form: Reject H_0 if $X_1^2 + X_2^2 + ...+X_n^2 \leq c$ for some $c > 0$

(D) If the most powerful test exists, it is of the form: Reject H_0 if $X_1^2 + X_2^2 + ...+ X_n^2 \geq c$ for some $c > 0$

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

Q. 41 – Q. 50 carry one marks each.

41. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+1}+\sqrt{n+2}+.....+\sqrt{n+n}}{n\sqrt{n}}$ (round off to 2 decimal places) equals __________.

42. Let f : [0, 2] → R be such that $|f(x) - f(y)| \le |x - y|^{4/3}$ for all x, y∈ [0, 2]. If $\int_0^2 f(x)dx = \dfrac{2}{3}$, then $\sum_{k=1}^{2019} f\left(\dfrac{1}{k}\right)$ equals___________.

43. The value (round off to 2 decimal places) of the double integral $\int\limits_0^9\int\limits_{\sqrt{x}}^3 \dfrac{1}{1+y^3}dydx$ equals-

44. If $\begin{bmatrix} \frac{\sqrt{5}}{3} & -\frac{2}{3} & c \\ \frac{2}{3} & \frac{\sqrt{5}}{3} & d \\ a & b & 1 \end{bmatrix}$ is a real orthogonal matrix, then $a^2 + b^2 + c^2 + d^2$ equals_______.

45. Two fair dice are tossed independently and it is found that one face is odd and the other one is even. Then the probability (round off to 2 decimal places) that the sum is less than 6 equals __________.

46. Let X be a random variable with the moment generating function

$$M_X(t) = \left(\frac{e^{\frac{t}{2}} + e^{-\frac{t}{2}}}{2}\right)^2, -\infty < t < \infty.$$

Using Chebyshev's inequality, the upper bound for $P\left(|X| > \sqrt{\frac{2}{3}}\right)$ equals ___________.

47. In a production line of a factory, each packet contains four items. Past record shows that 20% of the produced items are defective. A quality manager inspects each item in a packet and approves the packet for shipment if at most one item in the packet is found to be defective. Then the probability (round off to 2 decimal places) that out of the three randomly inspected packets at least two are approved for shipment equals __________.

48. Let X be the number of heads obtained in a sequence of 10 independent tosses of a fair coin. The fair coin is tossed again X number of times independently, and let Y be the number of heads obtained in these X number of tosses. Then E(X + 2Y) equals _________.

49. Let 0, 1, 0, 0, 1 be the observed values of a random sample of size five from a discrete distribution with the probability mass function $P(X = 1) = 1 - P(X = 0) = 1 - e^{-\lambda}$, where $\lambda > 0$. The method of moments estimate (round off to 2 decimal places) of λ equals _________.

50. Let X_1, X_2, X_3 be a random sample from $N(\mu_1, \sigma^2)$ distribution and Y_1, Y_2, Y_3 be a random sample from $N(\mu_2, \sigma^2)$ distribution. Also, assume that (X_1, X_2, X_3) and (Y_1, Y_2, Y_3) are independent. Let the observed values of $\sum_{i=1}^{3}\left[X_i - \frac{1}{3}(X_1 + X_2 + X_3)\right]^2$ and $\sum_{i=1}^{3}\left[Y_i - \frac{1}{3}(Y_1 + Y_2 + Y_3)\right]^2$ be 1 and 5, respectively. The length (round off to 2 decimal places) of the shortest 90% confidence interval of $\mu_1 - \mu_2$ equals _________.

Q. 51 – Q. 60 carry two marks each.

51. $\lim_{n\to\infty}\left[n - \frac{n}{e}\left(1+\frac{1}{n}\right)^n\right]$ equals __________.

52. For any real number y, let [y] be the greatest integer less than or equal to y and let {y} = y – [y]. For n = 1, 2,...., and for $x \in R$, let

$$f_{2n}(x) = \begin{cases} \left[\dfrac{\sin x}{x}\right], & x \neq 0 \\ 1, & x = 0 \end{cases} \quad \text{and } f_{2n-1}(x) = \begin{cases} \left\{\dfrac{\sin x}{x}\right\}, & x \neq 0 \\ 1, & x = 0 \end{cases}.$$

Then $\lim_{x\to 0}\sum_{k=1}^{100} f_k(x)$ equals _________.

53. The volume (round off to 2 decimal places) of the region in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 2$ and $y + z = 4$ equals _______.

54. If $ad - bc = 2$ and $ps - qr = 1$, then the determinant of $\begin{bmatrix} a & b & 0 & 0 \\ 3 & 10 & 2p & q \\ c & d & 0 & 0 \\ 2 & 7 & 2r & s \end{bmatrix}$ equals......

55. In an ethnic group, 30% of the adult male population is known to have heart disease. A test indicates high cholesterol level in 80% of adult males with heart disease. But the test also indicates high cholesterol levels in 10% of the adult males with no heart disease. Then the probability (round off to 2 decimal places), that a randomly selected adult male from this population does not have heart disease given that the test indicates high cholesterol level, equals......

56. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} ax^2, & 0 < x < 1 \\ bx^{-4}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

where a and b are positive real numbers. If E(X) = 1, then $E(X^2)$ equals.......

57. Let X and Y be jointly distributed continuous random variables, where Y is positive valued with $E(Y^2) = 6$. If the conditional distribution of X given Y = y is $U(1 - y, 1 + y)$, then Var(X) equals_____.

58. Let $X_1, X_2, \ldots, X_{10}$ be i.i.d N(0, 1) random variables. If $T = X_1^2 + X_2^2 + \ldots + X_{10}^2$, then $E\left(\frac{1}{T}\right)$ equals ___________.

59. Let X_1, X_2, X_3 be a random sample from a continuous distribution with the probability density function

$$f(x) = \begin{cases} e^{-(x-\mu)}, & x > \mu, \\ 0, & \text{otherwise} \end{cases}$$

Let $X_{(1)} = \min\{X_1, X_2, X_3\}$ and $c > 0$ be a real number. Then $(X_{(1)} - c, X_{(1)})$ is a 97% confidence interval for μ, if c (round off to 2-decimal places) equals........

60. Let X_1, X_2, X_3, X_4 be a random sample from a discrete distribution with the probability mass function $P(X = 0) = 1 - P(X = 1) = 1 - p$ for $0 < p < 1$. To test the hypothesis

$H_0 : p = 3/4$ against $H_1 : p = 4/5$,

consider the test:

Reject H_0 if $X_1 + X_2 + X_3 + X_4 > 3$.

Let the size and power of the test be denoted by α and γ, respectively. Then $\alpha + \gamma$ (round off to 2 decimal places) equals _________.

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
D	D	B	A	B	A	C	B	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	A	C	B	A	C	B	A	C	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	D	C	B	A	C	D	B	C	C

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
A,C	D	B,C	B,C,D	A,B,C,D	A,B	B,C,D	A,C,D	A,C	A,C

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	42	43	44	45	46	47	48	49	50
1.22	673	1.11	0	0.33	0.75	0.91	10	0.51	4.26
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
0.5	50	3.61	– 4	0.23	1.4	2	0.13	1.17	0.73

Mathematical Statistics (MS)

Previous Year Solved Paper 2018

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ),** wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 – Q.10 carry one mark each.

1. Let $\{a_n\}_{n\geq1}$ be a sequence of real numbers such that $a_1 = 2$ and, for $n \geq 1$,

$$a_{n+1} = \frac{2a_n + 1}{a_n + 1}$$

Then :

(A) $1.5 \leq a_n \leq 2$, for all natural number $n \geq 1$

(B) there exists a natural number $n \geq 1$ such that $a_n > 2$

(C) there exists a natural number $n \geq 1$ such that $a_n < 1.5$

(D) there exists a natural number $n \geq 1$ such that $a_n = \frac{1+\sqrt{5}}{2}$

2. The value of

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{n^2} e^{-2n}$$

is :

(A) e^{-2} (B) e^{-1}

(C) e (D) e^2

3. Let $\{a_n\}_{n\geq1}$ and $\{b_n\}_{n\geq1}$ be two convergent sequences of real numbers. For $n \geq 1$, define $u_n = \max\{a_n, b_n\}$ and $v_n = \min\{a_n, b_n\}$. Then

(A) neither $\{u_n\}_{n\geq1}$ nor $\{v_n\}_{n\geq1}$ converges

(B) $\{u_n\}_{n\geq1}$ converges but $\{v_n\}_{n\geq1}$ does not converge

(C) $\{u_n\}_{n\geq1}$ does not converge but $\{v_n\}_{n\geq1}$ converges

(D) both $\{u_n\}_{n\geq1}$ and $\{v_n\}_{n\geq1}$ converge

4. Let $M = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix}$. If I is the 2 × 2 identity matrix and 0 is the 2 × 2 zero matrix, then :

(A) $20\,M^2 - 13\,M + 7\,I = 0$ (B) $20\,M^2 - 13\,M + 7\,I = 0$

(C) $20\,M^2 + 13\,M + 7\,I = 0$ (D) $20\,M^2 + 13\,M - 7\,I = 0$

5. Let X be a random variable with the probability density function

$$f(x) = \begin{cases} \frac{a^p}{\Gamma(p)} e^{-ax} x^{p-1}, & x \geq 0,\ a > 0,\ p > 0, \\ 0, & \text{otherwise} \end{cases}$$

If E(X) = 20 and Var(X) = 10, then (a, p) is :

(A) (2, 20) (B) (2, 40)

(C) (4 20) (D) (4, 40)

6. Let X be a random variable with the distribution function

$$F(x)=\begin{cases}0, & x<0\\ \frac{1}{4}+\frac{4x-x^2}{8}, & 0\le x<2,\\ 1, & x\ge 2,\end{cases}$$

Then

$$P(X = 0) + P(X = 1.5) + P(X = 2) + P(X \ge 1)$$

equals :

(A) $\frac{3}{8}$ (B) $\frac{5}{8}$

(C) $\frac{7}{8}$ (D) 1

7. Let X_1, X_2 and X_3 be i.i.d. U(0, 1) random variables. Then $E\left(\frac{X_1+X_2}{X_1+X_2+X_3}\right)$ equals :

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

8. Let $x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $x_4 = 3$ and $x_5 = 0$ be the observed values of a random sample of size 5 from a discrete distribution with the probability mass function

$$f(x;\theta)=P(X=x)=\begin{cases}\frac{\theta}{3}, & x=0,\\ \frac{2\theta}{3}, & x=1,\\ \frac{1-\theta}{2}, & x=2,3,\end{cases}$$

where $\theta \in [0, 1]$ is the unknown parameter. Then the maximum likelihood estimate of θ is :

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$

(C) $\frac{5}{7}$ (D) $\frac{5}{9}$

9. Consider four coins labelled as 1, 2, 3 and 4. Suppose that the probability of obtaining a 'head' in a single toss of the i^{th} coin is $\frac{i}{4}$, i = 1, 2, 3, 4. A coin is chosen uniformly at random and flipped. Given that the flip resulted in a 'head', the conditional probability that the coin was labelled either 1 or 2 equals :

(A) $\frac{1}{10}$ (B) $\frac{2}{10}$

(C) $\frac{3}{10}$ (D) $\frac{4}{10}$

10. Consider the linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$; i = 1, 2, ..., n, where ϵ_i's are i.i.d. standard normal random variables. Given that

$$\frac{1}{n}\sum_{i=1}^{n} x_i = 3.2,\ \frac{1}{n}\sum_{i=1}^{n} y_i = 4.2,\ \frac{1}{n}\sum_{j=1}^{n}\left(x_j - \frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = 1.5 \text{ and}$$

$$\frac{1}{n}\sum_{ij=1}^{n}\left(x_j - \frac{1}{n}\sum_{i=1}^{n} x_i\right)\left(y_j - \frac{1}{n}\sum_{i=1}^{n} y_i\right) = 1.7$$

the maximum likelihood estimates of β_0 and β_1, respectively, are

(A) $\frac{17}{15}$ and $\frac{32}{75}$ (B) $\frac{32}{75}$ and $\frac{17}{15}$

(C) $\frac{17}{15}$ and $\frac{43}{75}$ (D) $\frac{43}{75}$ and $\frac{17}{15}$

Q. 11 – Q. 30 carry two marks each.

11. Let f:[– 1, 1] → $\mathbb{R}$ be defined by $f(x) = \frac{x^2 + [\sin \pi x]}{1 + |x|}$, where [y] denotes the greatest integer less than or equal to y. Then

(A) f is continuous at $-\frac{1}{2}, 0, 1$

(B) f is discontinuous at $-1, 0, \frac{1}{2}$

(C) f is discontinuous at $-1, -\frac{1}{2}, 0, \frac{1}{2}$

(D) f is continuous everywhere except at 0

12. Let f, g : $\mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - \frac{\cos x}{2}$ and $g(x) = \frac{x \sin x}{2}$. Then

(A) f(x) = g(x) for more than two values of x

(B) $f(x) \neq g(x)$, for all x in $\mathbb{R}$

(C) f(x) = g(x) for exactly one value of x

(D) f(x) = g(x) for exactly two values of x

13. Consider the domain $D = \{(x, y) \in \mathbb{R}^2 : x \leq y\}$ and the function $h : D \to \mathbb{R}$ defined by

$$h((x, y)) = (x - 2)^4 + (y - 1)^4,\ (x, y) \in D$$

Then the minimum value of h on D equals

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $\frac{1}{16}$

14. Let M = [X Y Z] be an orthogonal matrix with X, Y, Z $\in \mathbb{R}^3$ as its column vectors. Then $Q = XX^T + YY^T$

(A) is a skew-symmetric matrix
(B) is the 3 × 3 identity matrix
(C) satisfies $Q^2 = Q$
(D) satisfies QZ = Z

15. Let f:[0, 3] $\to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 0, & 0 \le x < 1, \\ e^{x^2} - e, & 1 \le x < 2, \\ e^{x^2} + 1, & 2 \le x \le 3, \end{cases}$$

Now, define F : [0, 3] $\to \mathbb{R}$ by

$$F(0) = 0 \text{ and } F(x) = \int_0^x f(t)\,dt \text{ for } 0 < x \le 3.$$

Then :

(A) F is differentiable at x = 1 and F'(1) = 0
(B) F is differentiable at x = 2 and F'(2) = 0
(C) F is not differentiable at x = 1
(D) F is differentiable at x = 2 and F'(2) = 1

16. If x, y and z are real numbers such that 4x + 2y + z = 31 and 2x + 4y – z = 19, then the value of 9x + 7y + z

(A) cannot be computed from the given information
(B) equals $\frac{281}{3}$
(C) equals $\frac{182}{3}$
(D) equals $\frac{218}{3}$

17. Let $M = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$. If

$$V = \left\{ (x, y, 0) \in \mathbb{R}^3 : M\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \text{ and } W = \left\{ (x, y, z) \in \mathbb{R}^3 : M\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\},$$

then

(A) the dimension of V equals 2
(B) the dimension of W equals 2
(C) the dimension of V equals 1
(D) $V \cap W = \{(0, 0, 0)\}$

18. Let M be a 3 × 3 non-zero, skew-symmetric real matrix. If I is the 3 × 3 identity matrix, then

(A) M is invertible
(B) the matrix I + M is invertible
(C) there exists a non-zero real number α such that $\alpha I + M$ is not invertible
(D) all the eigenvalues of M are real

19. Let X be a random variable with the moment generating function

$$M_X(t) = \frac{6}{\pi^2}\sum_{n\geq 1}\frac{e^{t^2/2n}}{n^2},\ t \in \mathbb{R}.$$

Then $P(X \in \mathbb{Q})$, where $\mathbb{Q}$ is the set of rational numbers, equals

(A) 0 (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

20. Let X be a discrete random variable with the moment generating function

$$M_X(t) = \frac{(1+3e^t)^2(3+e^t)^3}{1024},\ t \in \mathbb{R}.$$

Then :

(A) $E(X) = \frac{9}{4}$ (B) $Var(X) = \frac{15}{32}$

(C) $P(X \geq 1) = \frac{27}{1024}$ (D) $P(X = 5) = \frac{3}{1024}$

21. Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables with X_n having the probability density function as

$$f_n(x) = \begin{cases} \dfrac{1}{2^{n/2}\Gamma\left(\dfrac{n}{2}\right)} e^{-\frac{x}{2}} x^{\left(\frac{n}{2}-1\right)}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\lim_{n\to\infty}\left[P\left(X_n > \frac{3}{4}n\right) + P\left(X_n > n + 2\sqrt{2n}\right)\right]$$

equals

(A) $1 + \Phi(2)$ (B) $1 - \Phi(2)$

(C) $\Phi(2)$ (D) $2 - \Phi(2)$

22. Let X be a Poisson random variable with mean $\frac{1}{2}$. Then E((X+1)!) equals

(A) $2e^{-\frac{1}{2}}$ (B) $4e^{-\frac{1}{2}}$

(C) $4e^{-1}$ (D) $2e^{-1}$

23. Let X be a standard normal random variable. Then $P(X^3 - 2X^2 - X + 2 > 0)$ equals

(A) $2\Phi(1) - 1$ (B) $1 - \Phi(2)$

(C) $2\Phi(1) - \Phi(2)$ (D) $\Phi(2) - \Phi(1)$

24. Let X and Y have the joint probability density function

$$f(x, y) = \begin{cases} 2, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Let $a = E\left(Y \mid X = \frac{1}{2}\right)$ and $b = Var\left(Y \mid X = \frac{1}{2}\right)$. Then (a, b) is

(A) $\left(\frac{3}{4}, \frac{7}{12}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{48}\right)$

(C) $\left(\frac{1}{4}, \frac{7}{12}\right)$ (D) $\left(\frac{3}{4}, \frac{1}{48}\right)$

25. Let X and Y have the joint probability mass function

$$P(X = m, Y = n) = \begin{cases} \frac{m+n}{21}, & m = 1,2,3;\ n = 1,2, \\ 0, & \text{otherwise.} \end{cases}$$

Then P(X = 2|Y = 2) equals

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

26. Let X and Y be two independent standard normal random variables. Then the probability density function of $Z = \frac{|X|}{|Y|}$ is

(A) $f(z) = \begin{cases} \frac{\sqrt{1/2}}{\sqrt{\pi}} e^{-\frac{z}{2}} z^{-\frac{1}{2}}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$ (B) $f(z) = \begin{cases} \frac{2}{\sqrt{2\pi}} e^{-z^2/2}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$

(C) $f(z) = \begin{cases} e^{-z}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$ (D) $f(z) = \begin{cases} \frac{2}{\pi} \frac{1}{(1+z^2)}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$

27. Let X and Y have the joint probability density function

$$f(x,y) = \begin{cases} e^{-y}, & 0 < x < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

Then the correlation coefficient between X and Y equals

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{2}{\sqrt{3}}$

28. Let $x_1 = -2$, $x_2 = 1$ and $x_3 = -1$ be the observed values of a random sample of size three from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \dfrac{1}{2\theta+1}, & x \in \{-\theta, -\theta+1, ..., 0, ..., \theta\}, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \Theta = \{1, 2, ...\}$ is the unknown parameter. Then the method of moment estimate of θ is :

(A) 1 (B) 2

(C) 3 (D) 4

29. Let X be a random sample from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \dfrac{1}{\theta}, & x = 1, 2, ..., \theta, \\ 0, & \text{otherwise} \end{cases}$$

where $\theta \in \Theta = \{20, 40\}$ is the unknown parameter. Consider testing

$H_0 : \theta = 40$ against $H_1 : \theta = 20$

at a level of significance $\alpha = 0.1$.Then the uniformly most powerful test rejects H_0 if and only if

(A) $X \leq 4$ (B) $X > 4$

(C) $X \geq 3$ (D) $X < 3$

30. Let X_1 and X_2 be a random sample of size 2 from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \theta, & x = 0, \\ 1-\theta, & x = 1, \end{cases}$$

where $\theta \in \Theta = [0.2, 0.4]$ is the unknown parameter. For testing $H_0 : \theta = 0.2$ against $H_1 : \theta = 0.4$, consider a test with the critical region

$C = \{(x_1, x_2) \in \{0, 1\} \times \{0, 1\} : x_1 + x_2 < 2\}$.

Let α and β denote the probability of Type I error and power of the test, respectively.

Then (α, β) is :

(A) (0.36, 0.74) (B) (0.64, 0.36)

(C) (0.05, 0.64) (D) (0.36, 0.64)

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that

$$a_n = \sum_{k=n+1}^{2n} \frac{1}{k}, n \geq 1.$$

Then which of the following statement(s) is (are) true ?

(A) $\{a_n\}_{n\geq 1}$ is an increasing sequence

(B) $\{a_n\}_{n\geq 1}$ is bounded below

(C) $\{a_n\}_{n\geq 1}$ is bounded above

(D) $\{a_n\}_{n\geq 1}$ is an convergent sequence

32. Let $\sum_{n\geq1} a_n$ be a convergent series of positive real numbers. Then which of the following statement(s) is (are) true ?

(A) $\sum_{n\geq1} (a_n)^2$ is always convergent.

(B) $\sum_{n\geq1} \sqrt{a_n}$ is always convergent.

(C) $\sum_{n\geq1} \frac{\sqrt{a_n}}{n}$ is always convergent.

(D) $\sum_{n\geq1} \frac{\sqrt{a_n}}{n^{1/4}}$ is always convergent.

33. Let $\{a_n\}_{n\geq1}$ be a sequence of real numbers such that $a_1 = 3$ and for $n \geq 1$,

$$a_{n+1} = \frac{a_n^2 - 2a_n + 4}{2}$$

Then which of the following statement(s) is (are) true ?

(A) $\{a_n\}_{n\geq1}$ is a monotone sequence

(B) $\{a_n\}_{n\geq1}$ is a bounded sequence

(C) $\{a_n\}_{n^31}$ does not have finite limit, as $n \to \infty$

(D) $\lim_{n\to\infty} a_n = 2$

34. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^4\left(2 + \sin\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then which of the following statement(s) is (are) true ?

(A) f attains its minimum at 0

(B) f is monotone

(C) f is differentiable at 0

(D) $f(x) > 2x^4 + x^3$, for all $x > 0$

35. Let P be a probability function that assigns the same weight to each of the points of the sample space $\Omega = \{1,2,3,4\}$. Consider the events $E = \{1,2\}$, $F = \{1,3\}$ and $G = \{3,4\}$. Then which of the following statement(s) is (are) true ?

(A) E and F are independent

(B) E and G are independent

(C) F and G are independent

(D) E, F and G are independent

36. Let $X_1, X_2, ..., X_n$, $n \geq 5$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta, \\ 0, & \text{otherwise} \end{cases}$$

where $\theta \in \mathbb{R}$ is the unknown parameter. Then which of the following statement(s) is (are) true?

(A) A 95% confidence interval of θ has to be of finite length

(B) $(\min\{X_1, X_2, ..., X_n\} + \frac{1}{n}\ln(0.05), \min\{X_1, X_2, ..., X_n\})$ is a 95% confidence interval of θ

(C) A 95% confidence interval of θ can be of length 1

(D) A 95% confidence interval of θ can be of length 2

37. Let $X_1, X_2, ..., X_n$ be a random sample from $U(0, \theta)$, where $\theta > 0$ is the unknown parameter. Let $X_{(n)} = \max\{X_1, X_2, ..., X_n\}$. Then which of the following is (are) consistent estimator(s) of θ^3 ?

(A) $8X_n^3$ (B) $X_{(n)}^3$

(C) $\left(\frac{2}{n}\sum_{i=5}^{n} X_i\right)^3$ (D) $\frac{nX_{(n)}^3 + 1}{n+1}$

38. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} c(\theta)e^{-(x-\theta)}, & x \geq 2\theta, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \mathbb{R}$ is the unknown parameter. Then which of the following statement(s) is (are) true?

(A) The maximum likelihood estimator of θ is $\frac{\min\{X_1, X_2, ..., X_n\}}{2}$

(B) $c(\theta) = 1$, for all $\theta \in \mathbb{R}$

(C) The maximum likelihood estimator of θ is $\min\{X_1, X_2, ..., X_n\}$

(D) The maximum likelihood estimator of θ does not exist

39. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \theta^2 x e^{-\theta x}, & x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is the unknown parameter. If $Y = \sum_{i=1}^{n} X_i$, then which of the following statement(s) is (are) true ?

(A) Y is a complete sufficient statistic for θ

(B) $\frac{2n}{Y}$ is the uniformly minimum variance unbiased estimator of θ

(C) $\frac{2n-1}{Y}$ is the uniformly minimum variance unbiased estimator of θ

(D) $\frac{2n+1}{Y}$ is the uniformly minimum variance unbiased estimator of θ

40. Let $X_1, X_2, ..., X_n$ be a random sample from $U(\theta, \theta + 1)$, where $\theta \in \mathbb{R}$ is the unknown parameter. Let $U = \max\{X_1, X_2, ..., X_n\}$ and $V = \min\{X_1, X_2, ..., X_n\}$. Then which of the following statement(s) is (are) true ?

(A) U is a consistent estimator of θ

(B) V is a consistent estimator of θ

(C) $2U - V - 2$ is a consistent estimator of θ

(D) $2V - U + 1$ is a consistent estimator of θ

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that

$$a_n = \frac{1+3+5+...+(2n-1)}{n!}, n \geq 1.$$

Then $\sum_{n\geq 1} a_n$ converges to ________.

42. Let

$$S = \left\{(x, y) \in \mathbb{R}^2 : x, y \geq 0, \sqrt{4-(x-2)^2} \leq y \leq \sqrt{9-(x-3)^2}\right\}.$$

Then the area of S equals ___________.

43. Let S = $\{(x, y) \in \mathbf{R}^2 : |x| + |y| \leq 1\}$. Then the area of S equals _________.

44. Let

$$J = \frac{1}{\pi}\int_0^1 t^{-\frac{1}{2}}(1-t)^{\frac{3}{2}}\,dt.$$

Then the value of J equals ___________.

45. A fair die is rolled three times independently. Given that 6 appeared at least once, the conditional probability that 6 appeared exactly twice equals _________.

46. Let X and Y be two positive integer valued random variables with the joint probability mass function

$$P(X = m, Y = n) = \begin{cases} g(m)\,h(n), & m, n \geq 1, \\ 0, & \text{otherwise} \end{cases}$$

where $g(m) = \left(\frac{1}{2}\right)^{m-1}$, $m \geq 1$ and $h(n) = \left(\frac{1}{3}\right)^{n}$, $n \geq 1$. Then E(XY) equals ___________.

47. Let E, F and G be three events such that
$P(E \cap F \cap G) = 0.1$, $P(G \mid F) = 0.3$ and $P(E \mid F \cap G) = P(E \mid F)$.
Then $P(G \mid E \cap F)$ equals ________

48. Let A_1, A_2 and A_3 be three events such that

$$P(A_i) = \frac{1}{3}, i = 1, 2, 3;\ P(A_i \cap A_j) = \frac{1}{6}, 1 \leq i \neq j \leq 3 \text{ and } P(A_1 \cap A_2 \cap A_3) = \frac{1}{6}.$$

Then the probability that none of the events A_1, A_2, A_3 occur equals ___________.

49. Let $X_1, X_2, ..., X_n$ be a random sample from the distribution with the probability density function

$$f(x) = \frac{1}{4}e^{-|x-4|} + \frac{1}{4}e^{-|x-6|}, x \in \mathbb{R}$$

Then $\frac{1}{n}\sum_{i=1}^{n} X_i$ converges in probability to ___________

50. Let x_1 = 1.1, x_2 = 2.2 and x_3 = 3.3 be the observed values of a random sample of size three from a distribution with the probability density function

$$f(x;\theta)=\begin{cases}\frac{1}{\theta}e^{-x/\theta}, & x>0,\\ 0, & \text{otherwise,}\end{cases}$$

where $\theta \in \Theta$ = {1, 2, ...} is the unknown parameter. Then the maximum likelihood estimate of θ equals _________.

Q. 51 – Q. 60 carry two marks each.

51. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that f' is continuous on $\mathbb{R}$ with f'(3) = 18. Define

$$g_n(x)=n\left(f\left(x+\frac{5}{n}\right)-f\left(x-\frac{2}{n}\right)\right)$$

Then $\lim\limits_{n\to\infty} g_n(3)$ equals ___________.

52. Let M = $\sum_{i=1}^{4} X_i X_i^T$, where

$$X_1^T=[1\ \ -1\ \ 1\ \ 0],\ X_2^T=[1\ \ 1\ \ 0\ \ 1],\ X_3^T=[1\ \ 3\ \ 1\ \ 0] \text{ and } X_4^T=[1\ \ 1\ \ 1\ \ 0].$$

Then the rank of M equals ___________.

53. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $\lim\limits_{x\to\infty} f(x) = \infty$ and $\lim\limits_{x\to\infty} f'(x) = 2$. Then

$$\lim_{x\to\infty}\left(1+\frac{f(x)}{x^2}\right)^x$$

equals ________.

54. The value of

$$\int_0^{\frac{\pi}{2}}\left(\int_0^x e^{\sin y}\sin x\, dy\right)dx$$

equals ___________.

55. Let X be a random variable with the probability density function

$$f(x)=\begin{cases}4x^k, & 0<x<1,\\ x-\frac{x^2}{2}, & 1\le x<2,\\ 0, & \text{otherwise,}\end{cases}$$

where k is a positive integer. Then $P\left(\frac{1}{2}<X<\frac{3}{2}\right)$ equals _________.

56. Let X and Y be two discrete random variables with the joint moment generating function

$$M_{X,Y}(t_1,t_2)=\left(\frac{1}{3}e^{t_1}+\frac{2}{3}\right)^2\left(\frac{2}{3}e^{t_2}+\frac{1}{3}\right)^3,\ t_1,t_2\in\mathbb{R}$$

Then P(2X + 3Y > 1) equals __________.

57. Let X_1, X_2, X_3 and X_4 be i.i.d discrete random variables with the probability mass function

$$P(X_1 = n) = \begin{cases} \frac{3^{n-1}}{4^n}, & n = 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(X_1 + X_2 + X_3 + X_4 = 6)$ equals ___________.

58. Let X be a random variable with the probability mass function

$$P(X = n) = \begin{cases} \frac{1}{10}, & n = 1, 2, \ldots, 10. \\ 0, & \text{otherwise} \end{cases}$$

Then E(max{X, 5}) equals ____________.

59. Let X be a sample observation from $U(\theta, \theta^2)$ distribution, where $\theta \in \Theta = \{2, 3\}$ is the unknown parameter. For testing

$H_0 : \theta = 2$ against $H_1 : \theta = 3$

let α and β be the size and power, respectively, of the test that rejects H_0 if and only if $X \geq 3.5$. Then $\alpha + \beta$ equals __________.

60. A fair die is rolled four times independently. For i = 1, 2, 3, 4, define

$$Y_i = \begin{cases} 1, & \text{if 6 appears in the } i^{th} \text{ throw,} \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(\max\{Y_1, Y_2, Y_3, Y_4\} = 1)$ equals ___________.

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
A	A	D	B	B	C	C	B	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	D	C	C	A	D	C	B	A	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	B	C	D	A	D	C	B	A	D

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
A,B,C,D	A,C	A,C	A,C	A,C	B,C,D	B,C,D	A	A,C	B,C,D

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	42	43	44	45	46	47	48	49	50
5.40-5.50	7.85	2	0.375	0.1648	3	0.3	0.3	5	2
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
126	3	7.39	1.71	0.8854	0.9835	0.02197	6.5	1.167	0.5177

Mathematical Statistics (MS)

Previous Year Solved Paper 2017

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains **Numerical Answer Type Questions (NAT).** For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ),** wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.

Special Instructions/Useful Data

$\mathbb{N}$	Set of all natural numbers
$\mathbb{Q}$	Set of all rational numbers
$\mathbb{R}$	Set of all real numbers
P^T	Transpose of the matrix P
$\mathbb{R}^n$	$\{(x_1, x_2, ..., x_n)^T \mid x_i \in \mathbb{R}, i = 1, 2, ... , n\}$
g'	Derivative of a real valued function g
g"	Second derivative of a real valued function g
P(A)	Probability of an event A
i.i.d.	Independently and identically distributed
$N(\mu,\sigma^2)$	Normal distribution with mean m and variance σ^2
$F_{m,n}$	F distribution with (m, n) degrees of freedom
t_n	Student's t distribution with n degrees of freedom
χ_n^2	Central Chi-squared distribution with n degrees of freedom
$\Phi(x)$	Cumulative distribution function of N(0, 1)
A^C	Complement of a set A
E(X)	Expectation of a random variable X
Var(X)	Variance of a random variable X
Cov(X,Y)	Covariance between random variables X and Y
r!	Factorial of an integer r > 0, 0! = 1

$$\Phi(0.25) = 0.5987,\ \Phi(0.5) = 0.6915,\ \Phi(0.625) = 0.7341,\ \Phi(0.71) = 0.7612,$$
$$\Phi(1) = 0.8413,\ \Phi(1.125) = 0.8697,\ \Phi(1.5) = 0.9332,\ \Phi(1.64) = 0.95,$$
$$\Phi(2) = 0.9772$$

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 to Q.10 carry one mark each.

1. The imaginary parts of the eigenvalues of the matrix

$$P = \begin{pmatrix} 3 & 2 & 5 \\ 2 & -3 & 6 \\ 0 & 0 & -3 \end{pmatrix}$$

are

(A) 0, 0, 0 (B) 2, –2, 0

(C) 1, –1, 0 (D) 3, –3, 0

2. Let u, v $\in \mathbb{R}^4$ be such that u = $(1 \ \ 2 \ \ 3 \ \ 5)^T$ and v = $(5 \ \ 3 \ \ 2 \ \ 1)^T$. Then the equation $uv^T x = v$ has

(A) infinitely many solutions (B) no solution

(C) exactly one solution (D) exactly two solutions

3. Let $u_n = \left(4 - \frac{1}{n}\right)^{\frac{(-1)^n}{n}}$, $n \in \mathbb{N}$ and let $l = \lim_{n\to\infty} u_n$.

Which of the following statements is TRUE ?

(A) $l = 0$ and $\sum_{n=1}^{\infty} u_n$ is convergent (B) $l = \frac{1}{4}$ and $\sum_{n=1}^{\infty} u_n$ is divergent

(C) $l = \frac{1}{4}$ and $\{u_n\}_{n\ge1}$ is oscillatory (D) $l = 1$ and $\sum_{n=1}^{\infty} u_n$ is divergent

4. Let $\{a_n\}_{n\ge1}$ be a sequence defined as follows :

$$a_1 = 1 \text{ and } a_{n+1} = \frac{7a_n + 11}{21}, \ n \in \mathbb{N}.$$

Which of the following statements is TRUE ?

(A) $\{a_n\}_{n\ge1}$ is an increasing sequence which diverges

(B) $\{a_n\}_{n\ge1}$ is an increasing sequence with $\lim_{n\to\infty} a_n = \frac{11}{14}$

(C) $\{a_n\}_{n\ge1}$ is a decreasing sequence which diverges

(D) $\{a_n\}_{n\ge1}$ is a decreasing sequence with $\lim_{n\to\infty} a_n = \frac{11}{14}$

5. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} 0, & \text{if } x \le 0 \\ x^3, & \text{if } 0 < x \le 1 \\ \frac{3}{x^5}, & \text{if } x > 1 \end{cases}$$

The $P\left(\frac{1}{2} < X < 2\right)$ equals

(A) $\frac{15}{16}$ (B) $\frac{11}{16}$

(C) $\frac{7}{12}$ (D) $\frac{3}{8}$

6. Let X be a random variable with the moment generating function

$$M_X(t) = \frac{1}{216}(5 + e^t)^3,\ t \in \mathbb{R}.$$

Then P(X > 1) equals

(A) $\frac{2}{27}$ (B) $\frac{1}{27}$

(C) $\frac{1}{12}$ (D) $\frac{2}{9}$

7. Let X be a discrete random variable with the probability mass function

$p(x) = k(1 + |x|)^2$, x = –2, –1, 0, 1, 2,

where k is a real constant. Then P(X = 0) equals

(A) $\frac{1}{9}$ (B) $\frac{2}{27}$

(C) $\frac{1}{27}$ (D) $\frac{1}{81}$

8. Let the random variable X have uniform distribution on the interval $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$. Then P(cos X > sin X) is

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

9. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables having common probability density function

$$f(x) = \begin{cases} xe^{-x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, n = 1, 2, ... Then $\lim_{n\to\infty} P(\bar{X}_n = 2)$ equals

(A) 0 (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) 1

10. Let X_1, X_2, X_3 be a random sample from a distribution with the probability density function

$$f(x\,|\,\theta) = \begin{cases} \frac{1}{\theta}e^{-\frac{x}{\theta}}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}, \quad \theta > 0$$

Which of the following estimators of θ has the smallest variance for all $\theta > 0$?

(A) $\frac{X_1 + 3X_2 + X_3}{5}$ (B) $\frac{X_1 + X_2 + 2X_3}{4}$

(C) $\frac{X_1 + X_2 + X_3}{3}$ (D) $\frac{X_1 + 2X_2 + 3X_3}{6}$

Q.11-30 carry two marks each.

11. Player P_1 tosses 4 fair coins and player P_2 tosses a fair die independently of P_1. The probability that the number of heads observed is more than the number on the upper face of the die, equals

(A) $\frac{7}{16}$ (B) $\frac{5}{32}$

(C) $\frac{17}{96}$ (D) $\frac{21}{64}$

12. Let X_1 and X_2 be i.i.d. continuous random variables with the probability density function

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Using Chebyshev's inequality, the lower bound of $P\left(|X_1 + X_2 - 1| \leq \frac{1}{2}\right)$ is

(A) $\frac{5}{6}$ (B) $\frac{4}{5}$

(C) $\frac{3}{5}$ (D) $\frac{1}{3}$

13. Let X_1, X_2, X_3 be i.i.d. discrete random variables with the probability mass function

$$p(k) = \left(\frac{2}{3}\right)^{k-1}\left(\frac{1}{3}\right), \; k = 1, 2, 3, \dots$$

Let $Y = X_1 + X_2 + X_3$. Then $P(Y \geq 5)$ equals

(A) $\frac{1}{9}$ (B) $\frac{8}{9}$

(C) $\frac{2}{27}$ (D) $\frac{25}{27}$

14. Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} cx(1-x), & \text{if } 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases},$$

where c is a positive real constant. Then E(X) equals

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$

(C) $\frac{2}{5}$ (D) $\frac{1}{3}$

15. Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} x + y, & \text{if } 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then $P\left(X + Y > \frac{1}{2}\right)$ equals

(A) $\frac{23}{24}$ (B) $\frac{1}{12}$

(C) $\frac{11}{12}$ (D) $\frac{1}{24}$

16. Let $X_1, X_2, \dots, X_m, Y_1, Y_2, \dots, Y_n$ be i.i.d. N(0, 1) random variables. Then

$$W = \frac{n\left(\sum_{i=1}^{m} X_i\right)^2}{m\left(\sum_{j=1}^{n} Y_j^2\right)}$$

has

(A) χ^2_{m+n} distribution (B) t_n distribution

(C) $F_{m,n}$ distribution (D) $F_{1,n}$ distribution

17. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables with the probability mass function

$$f(x) = \begin{cases} \frac{1}{4}, & \text{if } x = 4 \\ \frac{3}{4}, & \text{if } x = 8 \\ 0, & \text{otherwise} \end{cases}$$

Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, n = 1, 2, ... If $\lim_{n\to\infty} P(m \leq \bar{X}_n \leq M) = 1$, then possible values of m and M are

(A) m = 2.1, M = 3.1 (B) m = 3.2, M = 4.1

(C) m = 4.2, M = 5.7 (D) m = 6.1, M = 7.1

18. Let x_1 = 1.1, x_2 = 0.5, x_3 = 1.4, x_4 = 1.2 be the observed values of a random sample of size four from a distribution with the probability density function

$$f(x \mid \theta) = \begin{cases} e^{\theta - x}, & \text{if } x \geq \theta \\ 0, & \text{otherwise} \end{cases}, \ \theta \in (-\infty, \infty).$$

Then the maximum likelihood estimate of θ^2 is

(A) 0.5 (B) 0.25

(C) 1.21 (D) 1.44

19. Let x_1 = 2, x_2 = 1, x_3 = $\sqrt{5}$, x_4 = $\sqrt{2}$ be the observed values of a random sample of size four from a distribution with the probability density function

$$f(x \mid \theta) = \begin{cases} \frac{1}{2\theta}, & \text{if } -\theta \leq x \leq \theta \\ 0, & \text{otherwise} \end{cases}, \ \theta > 0$$

Then the method of moments estimate of θ is

(A) 1 (B) 2

(C) 3 (D) 4

20. Let X_1, X_2 be a random sample from an N(0, θ) distribution, where θ > 0. Then the value of k, for which the interval $\left(0, \frac{X_1^2 + X_2^2}{k}\right)$ is a 95% confidence interval for θ, equals

(A) $-\log_e (0.95)$ (B) $-2 \log_e (0.95)$

(C) $-\frac{1}{2}\log_e (0.95)$ (D) 2

21. Let X_1, X_2, X_3, X_4 be a random sample from $N(\theta_1, \sigma^2)$ distribution and Y_1, Y_2, Y_3, Y_4 be a random sample from $N(\theta_2, \sigma^2)$ distribution, where $\theta_1, \theta_2 \in (-\infty, \infty)$ and $\sigma > 0$. Further suppose that the two random samples are independent. For testing the null hypothesis $H_0 : \theta_1 = \theta_2$ against the alternative hypothesis $H_1 : \theta_1 > \theta_2$, suppose that a test ψ rejects H_0 if and only if $\sum_{i=1}^{4} X_i > \sum_{j=1}^{4} Y_j$. The power of the test ψ at $\theta_1 = 1 + \sqrt{2}$, $\theta_2 = 1$ and $\sigma^2 = 4$ is

(A) 0.5987 (B) 0.7341

(C) 0.7612 (D) 0.8413

22. Let X be a random variable having a probability density function $f \in \{f_0, f_1\}$, where

$$f_0(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

and $$f_1(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

For testing the null hypothesis $H_0 : f \equiv f_0$ against $H_1 : f \equiv f_1$, based on a single observation on X, the power of the most powerful test of size $\alpha = 0.05$ equals

(A) 0.425 (B) 0.525

(C) 0.625 (D) 0.725

23. If $\int_{y=0}^{1}\int_{x=y}^{2-\sqrt{1-(y-1)^2}} f(x,y)dx\,dy = \int_{x=0}^{1}\int_{y=0}^{\alpha(x)} f(x,y)dy\,dx + \int_{x=1}^{2}\int_{y=0}^{\beta(x)} f(x,y)dy\,dx,$ then $\alpha(x)$ and $\beta(x)$ are

(A) $\alpha(x) = x,\ \beta(x) = 1 + \sqrt{1-(x-2)^2}$ (B) $\alpha(x) = x,\ \beta(x) = 1 - \sqrt{1-(x-2)^2}$

(C) $\alpha(x) = 1 + \sqrt{1-(x-2)^2},\ \beta(x) = x$ (D) $\alpha(x) = 1 - \sqrt{1-(x-2)^2},\ \beta(x) = x$

24. Let $f : [0, 1] \to \mathbb{R}$ be a function defined as

$$f(t) = \begin{cases} t^3\left(1+\frac{1}{5}\cos(\log_e t^4)\right) & \text{if } t \in (0,1] \\ 0 & \text{if } t = 0 \end{cases}$$

Let $F : [0, 1] \to \mathbb{R}$ be a defined as

$$F(x) = \int_0^x f(t)dt$$

Then F”(0) equals

(A) 0 (B) $\frac{3}{5}$

(C) $-\frac{5}{3}$ (D) $\frac{1}{5}$

25. Consider the function

$f(x, y) = x^3 - y^3 - 3x^2 + 3y^2 + 7,\ x, y \in \mathbb{R}.$

Then the local minimum (m) and the local maximum (M) of f are given by

(A) m = 3, M = 7 (B) m = 4, M = 11

(C) m = 7, M = 11 (D) m = 3, M = 11

26. For $c \in \mathbb{R}$, let the sequence $\{u_n\}_{n\ge1}$ be defined by

$$u_n = \frac{\left(1+\frac{c}{n}\right)^{n^2}}{\left(3-\frac{1}{n}\right)^{n}}$$

Then the values of c for which the series $\sum_{n=1}^{\infty} u_n$ converges are

(A) $\log_e 6 < c < \log_e 9$ (B) $c < \log_e 3$

(C) $\log_e 9 < c < \log_e 12$ (D) $\log_e 3 < c < \log_e 6$

27. If for a suitable $\alpha > 0$,

$$\lim_{x \to 0}\left(\frac{1}{e^{2x}-1}-\frac{1}{\alpha x}\right)$$

exists and is equal to l ($|l| < \infty$), then

(A) $\alpha = 2,\ l = 2$

(B) $\alpha = 2,\ l = -\frac{1}{2}$

(C) $\alpha = \frac{1}{2},\ l = -2$

(D) $\alpha = \frac{1}{2},\ l = -\frac{1}{2}$

28. Let

$$P = \int_0^1 \frac{dx}{\sqrt{8-x^2-x^3}}$$

Which of the following statements is TRUE ?

(A) $\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right) < P < \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{2}\right)$

(B) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{2}\right) < P < \sin^{-1}\left(\frac{1}{2}\right)$

(C) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right) < P < \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)$

(D) $\sin^{-1}\left(\frac{1}{2}\right) < P < \frac{\sqrt{3}}{2}\sin^{-1}\left(\frac{1}{2}\right)$

29. Let Q, A, B be matrices of order $n \times n$ with real entries such that Q is orthogonal and A is invertible. Then the eigenvalues of $Q^T A^{-1} BQ$ are always the same as those of

(A) AB

(B) $Q^T A^{-1} B$

(C) $A^{-1} BQ^T$

(D) BA^{-1}

30. Let $(x(t), y(t))$, $1 \le t \le \pi$, be the curve defined by

$$x(t) = \int_1^t \frac{\cos z}{z^2}dz \quad \text{and} \quad y(t) = \int_1^t \frac{\sin z}{z^2}dz$$

Let L be the length of the arc of this curve from the origin to the point P on the curve at which the tangent is perpendicular to the x-axis. Then L equals

(A) $\sqrt{2}$

(B) $\frac{\pi}{\sqrt{2}}$

(C) $1-\frac{2}{\pi}$

(D) $\frac{\pi}{2}+\sqrt{2}$

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

Q.31 to Q.40 carry two marks each.

31. Let $v \in \mathbb{R}^k$ with $v^T v \neq 0$. Let

$$P = I - 2\frac{vv^T}{v^Tv},$$

where I is the k × k identity matrix. Then which of the following statements is (are) TRUE ?

(A) $P^{-1} = I - P$

(B) -1 and 1 are eigenvalues of P

(C) $P^{-1} = P$

(D) $(I + P)v = v$

32. Let $\{a_n\}_{n\geq1}$ and $\{b_n\}_{n\geq1}$ be sequence of real numbers such that $\{a_n\}_{n\geq1}$ is increasing and $\{b_n\}_{n\geq1}$ is decreasing. Under which of the following conditions, the sequence $\{a_n + b_n\}_{n\geq1}$ is always convergent ?

(A) $\{a_n\}_{n\geq1}$ and $\{b_n\}_{n\geq1}$ are bounded sequences

(B) $\{a_n\}_{n\geq1}$ is bounded above

(C) $\{a_n\}_{n\geq1}$ is bounded above and $\{b_n\}_{n\geq1}$ is bounded below

(D) $a_n \to \infty$ $b_n \to -\infty$

33. Let $f : [0, 1] \to [0, 1]$ be defined as follows :

$$f(x) = \begin{cases} x, & \text{if } x \in \mathbb{Q} \cap [0,1] \\ x + \frac{2}{3}, & \text{if } x \in \mathbb{Q}^C \cap \left(0, \frac{1}{3}\right) \\ x - \frac{1}{3}, & \text{if } x \in \mathbb{Q}^C \cap \left(\frac{1}{3}, 1\right) \end{cases}$$

Which of the following statements is (are) TRUE ?

(A) f is one-one and onto

(B) f is not one-one but onto

(C) f is continuous on $\mathbb{Q} \cap [0, 1]$

(D) f is discontinuous everywhere on [0, 1]

34. Let f(x) be a non-negative differentiable function on $[a, b] \subset \mathbb{R}$ such that f(a) = 0 = f(b) and $|f'(x)| \leq 4$. Let L_1 and L_2 be the straight lines given by the equations $y = 4(x - a)$ and $y = -4(x - b)$, respectively. Then which of the following statements is (are TRUE) ?

(A) The curve y = f(x) will always lie below the lines L_1 and L_2

(B) The curve y = f(x) will always lie above the lines L_1 and L_2

(C) $\left|\int_a^b f(x)dx\right| < (b-a)^2$

(D) The point of intersection of the lines L_1 and L_2 lie on the curve y = f(x)

35. Let E and F be two events with $0 < P(E) < 1$, $0 < P(F) < 1$ and $P(E) + P(F) \geq 1$. Which of the following statements is (are) TRUE ?

(A) $P(E^C) \leq P(F)$

(B) $P(E \cup F) < P(E^C \cup F^C)$

(C) $P(E \mid F^C) \geq P(F^C \mid E)$

(D) $P(E^C \mid F) \leq P(F \mid E^C)$

36. The cumulative distribution function of a random variable X is given by

$$F(x)=\begin{cases}0, & \text{if } x<0\\ \frac{4}{9}, & \text{if } 0\le x<1\\ \frac{8}{9}, & \text{if } 1\le x<2\\ 1, & \text{if } x\ge 2\end{cases}$$

Which of the following statements is (are) TRUE ?

(A) The random variable X takes positive probability only at two points

(B) $P(1 \le X \le 2) = \frac{5}{9}$

(C) $E(X) = \frac{2}{3}$

(D) $P(0 < X < 1) = \frac{4}{9}$

37. Let X_1, X_2 be a random sample from a distribution with the probability mass function

$$f(x\mid\theta)=\begin{cases}1-\theta, & \text{if } x=0\\ \theta, & \text{if } x=1\\ 0, & \text{otherwise}\end{cases},\quad 0<\theta<1.$$

Which of the following is (are) unbiased estimator(s) of θ?

(A) $\frac{X_1+X_2}{2}$ (B) $\frac{X_1^2+X_2}{2}$

(C) $\frac{X_1^2+X_2^2}{2}$ (D) $\frac{X_1+X_2-X_1^2}{2}$

38. Let X_1, X_2, X_3 be a random sample from a distribution with the probability density function

$$f(x\mid\theta)=\begin{cases}\frac{1}{\theta}e^{-x/\theta}, & \text{if } x>0\\ 0, & \text{otherwise}\end{cases},\quad \theta>0.$$

If $\delta(X_1, X_2, X_3)$ is an unbiased estimator of θ, which of the following CANNOT be attained as a value of the variance of δ at $\theta = 1$?

(A) 0.1 (B) 0.2

(C) 0.3 (D) 0.5

39. Let X_1, X_2, ..., X_n ($n \ge 2$) be a random sample from a distribution with the probability density function

$$f(x\mid\theta)=\begin{cases}\frac{x}{\theta^2}e^{-x/\theta}, & \text{if } x>0\\ 0, & \text{otherwise}\end{cases},\quad \theta>0.$$

Let $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$. Which of the following statistics is (are) sufficient but NOT complete ?

(A) $\bar{X}$ (B) $\bar{X}^2+3$

(C) $(X_1, \Sigma_{i=2}^{n}X_i)$ (D) $(X_1, \bar{X})$

40. Let X_1, X_2, X_3, X_4 be a random sample from an $N(\theta, 1)$ distribution, where $\theta \in (-\infty, \infty)$. Suppose the null hypothesis $H_0 : \theta = 1$ is to be tested against the hypothesis $H_1 : \theta < 1$ at $\alpha = 0.05$ level of significance. For what observed values of $\Sigma_{i=1}^{4} X_i$, the uniformly most powerful test would reject H_0 ?

(A) -1 (B) 0

(C) 0.5 (D) 0.8

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

Q.41 to Q.50 carry one mark each.

41. Let the random variable X have uniform distribution on the interval (0, 1) and $Y = -2 \log_e X$. Then E(Y) equals ________.

42. If $Y = \log_{10} X$ has $N(\mu, \sigma^2)$ distribution with moment generating function $M_Y(t) = e^{5t+2t^2}$, $t \in (-\infty, \infty)$, then $P(X < 1000)$ equals ________.

43. Let X_1, X_2, X_3, X_4, X_5 be independent random variables with $X_1 \sim N(200, 8)$, $X_2 \sim N(104, 8)$, $X_3 \sim N(108, 15)$, $X_4 \sim N(120, 15)$ and $X_5 \sim N(210, 15)$. Let $U = \frac{X_1 + X_2}{2}$ and $V = \frac{X_3 + X_4 + X_5}{3}$.

Then $P(U > V)$ equals _________.

44. Let X and Y be discrete random variables with the joint probability mass function.

$$p(x, y) = \frac{1}{25}(x^2 + y^2), \text{ if } x = 1, 2;\ y = 0, 1, 2.$$

Then $P(Y = 1 \mid X = 1)$ equals _________.

45. Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then 9Cov(X, Y) equals __________.

46. Let $X_1, X_2, X_3, Y_1, Y_2, Y_3, Y_4$ be i.i.d. $N(\mu, \sigma^2)$ random variables. Let $\bar{X} = \frac{1}{3}\sum_{i=1}^{3} X_i$ and $\bar{Y} = \frac{1}{4}\sum_{j=1}^{4} Y_j$.

If $k\sqrt{\frac{15}{7}} \frac{(\bar{X} - \bar{Y})}{\sqrt{\left\{\Sigma_{i=1}^{3}(X_i - \bar{X})^2 + \Sigma_{i=1}^{4}(Y_j - \bar{Y})^2\right\}}}$ has t_v distribution, then $(v - k)$ equals __________.

47. Let $f : \left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be defined as

$$f(x) = \alpha x + \beta \sin x,$$

where $\alpha, \beta \in \mathbb{R}$. Let f have a local minimum at $x = \frac{\pi}{4}$ with $f\left(\frac{\pi}{4}\right) = \frac{\pi - 4}{4\sqrt{2}}$.

Then $8\sqrt{2}\,\alpha + 4\beta$ equals __________.

48. The area bounded between two parabolas $y = x^2 + 4$ and $y = -x^2 + 6$ is __________.

49. For $j = 1, 2, ..., 5$, let P_j be the matrix of order 5×5 obtained by replacing the j^{th} column of the identity matrix of order 5×5 with the column vector $v = (5\ \ 4\ \ 3\ \ 2\ \ 1)^T$. Then the determinant of the matrix product $P_1 P_2 P_3 P_4 P_5$ is __________.

50. Let $u_n = \dfrac{18n+3}{(3n-1)^2(3n+2)^2}$, $n \in \mathbb{N}$. Then $\Sigma_{n=1}^{\infty} u_n$ equals __________.

Q.51 to Q.60 carry two marks each.

51. Let a unit vector $v = (v_1\ \ v_2\ \ v_3)^T$ be such that $Av = 0$ where

$$A = \begin{pmatrix} \frac{5}{6} & -\frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{6} & -\frac{1}{3} & \frac{5}{6} \end{pmatrix}$$

Then the value of $\sqrt{6}\,(|v_1| + |v_2| + |v_3|)$ equals __________.

52. Let $F(x) = \int_0^x e^t(t^2 - 3t - 5)dt$, $x > 0$. Then the number of roots of $F(x) = 0$ in the interval $(0, 4)$ is __________.

53. A tangent is drawn on the curve $y = \frac{1}{3}\sqrt{x^3}$, $(x > 0)$ at the point $P\left(1, \frac{1}{3}\right)$ which meets the x-axis at Q. Then the length of the closed curve OQPO, where O is the origin, is __________.

54. The volume of the region

$$R = \{(x, y, z) \in \mathbb{R}^3 : x + y + z \le 3,\ y^2 \le 4x,\ 0 \le x \le 1,\ y \ge 0,\ z \ge 0\}$$

is __________.

55. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x}{8}, & \text{if } 0 < x < 2 \\ \frac{k}{8}, & \text{if } 2 \le x \le 4 \\ \frac{6-x}{8}, & \text{if } 4 < x < 6 \\ 0, & \text{otherwise} \end{cases}$$

where k is a real constant. Then $P(1 < X < 5)$ equals __________.

56. Let X_1, X_2, X_3 be independent random variables with the common probability density function

$$f(x) = \begin{cases} 2e^{-2x}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Let Y = min $\{X_1, X_2, X_3\}$, E(Y) = μ_y and Var(Y) = σ_y^2. Then $P(Y > \mu_y + \sigma_y)$ equals __________.

57. Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{2}e^{-x}, & \text{if } |y| \le x,\ x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Then E(X | Y = –1) equals __________.

58. Let X and Y be discrete random variables with $P(Y \le \{0, 1\}) = 1$,

$$P(X = 0) = \frac{3}{4}, \qquad P(X = 1) = \frac{1}{4},$$

$$P(Y = 1 | X = 1) = \frac{3}{4}, \quad P(Y = 0 | X = 0) = \frac{7}{8}.$$

Then 3P(Y = 1) – P(Y = 0) equals __________.

59. Let $X_1, X_2, ..., X_{100}$ be i.i.d. random variables with $E(X_1) = 0$, $E(X_1^2) = \sigma^2$, where $\sigma > 0$. Let $S = \Sigma_{i=1}^{100} X_i$. If an approximate value of $P(S \le 30)$ is 0.9332, then σ^2 equals __________.

60. Let X be a random variable with the probability density function

$$f(x | r, \lambda) = \frac{\lambda^r}{(r-1)!} x^{r-1} e^{-\lambda x},\ x > 0,\ \lambda > 0,\ r > 0.$$

If E(X) = 2 and Var(X) = 2, then P(X < 1) equals __________.

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
A	B	D	D	A	A	C	D	A	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	C	B	C	A	D	D	B	C	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	B	B	A	D	B	B	A	D	X

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
B,C	A,C	A,D	A,C	A,C,D	C,D	A,B,C	A,B,C	C,D	A,B,C

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	42	43	44	45	46	47	48	49	50
2.0	0.15	0.97	0.25	0.16	3.0	4	2.66	120	0.25
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
4	0	0.018	2.1	0.875	0.13	2	0.125	4.0	0.26

Mathematical Statistics (MS)

Previous Year Solved Paper 2016

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains **Numerical Answer Type Questions (NAT).** For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ),** wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.

Special Instructions / User Data	
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^n$	$\{(x_1,\dots,x_n) : x_i \in \mathbb{R}, i = 1, \dots, n\}$
P (A)	Probability of an event A
i.i.d.	Independently and identically distributed
Bin (n, p)	Binomial distribution with parameters n and p
poission (θ)	Poission distribution with mean θ
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
Exp (λ)	The exponential distribution with probability density function $f(x \mid \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}, \ \lambda > 0$
t_n	Student's t distribution with n degrees of freedom
χ_n^2	Chi-square distribution with n degrees of freedom
$\chi_{n,a}^2$	A constant such that $P(W > \chi_{n,a}^2) = \alpha$, where W has χ_n^2 distribution
$\Phi(x)$	Cummulative distribution function of N (0, 1)
$\phi(x)$	Probability density function of N (0,1)
A^c	Complement of an event A
$E(X)$	Expectation of random variable X
Var (X)	Variance of a random variable X
$B(m, n)$	$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx, m > 0, n > 0$
$[x]$	The greatest integer less than or equal to real number x
f'	Derivatives of function f
$\Phi(0.25) = 0.5987, \Phi(0.5) = 0.6915, \Phi(0.625) = 0.7341, \Phi(0.71) = 0.7612,$ $\Phi(1) = 0.8413, \Phi(1.125) = 0.8697, \Phi(2) = 0.9772$	

SECTION – A MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 to Q.10 carry one marks each.

1. Let

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ -1 & -2 & 1 & 1 \\ 1 & 2 & -3 & -7 \\ 1 & 2 & -2 & -4 \end{bmatrix}$$

Then rank of P equals

(A) 4 (B) 3

(C) 2 (D) 1

2. Let α, β, γ be real numbers such that $\beta \neq 0$ and $\gamma \neq 0$. Suppose

$$P = \begin{bmatrix} \alpha & \beta \\ \gamma & 0 \end{bmatrix}$$

and $P^{-1} = P$. Then

(A) $\alpha = 0$ and $\beta\gamma = 1$ (B) $\alpha \neq 0$ and $\beta\gamma = 1$

(C) $\alpha = 0$ and $\beta\gamma = 2$ (D) $\alpha = 0$ and $\beta\gamma = -1$

3. Let m > 1. The volume of the solid generated by revolving the region between the y–axis and the curve xy = 4, $1 \leq y \leq m$, about the y–axis is 15π. The value of m is

(A) 14 (B) 15

(C) 16 (D) 17

4. Consider the region S enclosed by the surface $z = y^2$ and the planes $z = 1$, $x = 0$, $x = 1$, $y = -1$ and $y = 1$. The volume of S is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) 1 (D) $\frac{4}{3}$

5. Let X be a discrete random variable with the moment generating function

$$M_x(t) = e^{0.5(e^t - 1)}, \; t \in \mathbb{R}$$

Then $P(X \leq 1)$ equals

(A) $e^{-1/2}$ (B) $\frac{3}{2} e^{-1/2}$

(C) $\frac{1}{2} e^{-1/2}$ (D) $e^{-(e-1)/2}$

6. Let E and F be two independent events with

$P(E \mid F) + P(F \mid E) = 1,\ P(E \cap F) = \frac{2}{9}$ and $P(F) < P(E)$.

Then $P(E)$ equals

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

7. Let X be a continuous random variable with the probability density function

$$f(x) = \frac{1}{(2 + x^2)^{3/2}},\ x \in \mathbb{R}.$$

Then $E(X^2)$

(A) equals 0 (B) equals 1

(C) equals 2 (D) does not exist

8. The probability density function of a random variable X is given by

$$f(x) = \begin{cases} \alpha x^{\alpha-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases},\ \alpha > 0$$

Then the distribution of the random variable $Y = \log_e X^{-2\alpha}$ is

(A) χ_2^2 (B) $\frac{1}{2}\chi_2^2$

(C) $2\chi_2^2$ (D) χ_1^2

9. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. $N(0,1)$ random variables.Then, as $n \to \infty$, $-\sum$

Q.11 – Q.30 carry two marks each.

11. Let (X, Y) have the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{2} y^2 e^{-x}, & \text{if } 0 < y < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

Then $P(Y < 1 \mid X = 3)$ equals

(A) $\frac{1}{81}$ (B) $\frac{1}{27}$

(C) $\frac{1}{9}$ (D) $\frac{1}{3}$

12. Let $X_1, X_2, ...$ be a sequence of i.i.d. random variables having the probability density function

$$f(x) = \begin{cases} \frac{1}{B(6,4)} x^5 (1-x)^3, & 0 < x < 1, \\ 0, & \text{otherwise} \end{cases}$$

Let $Y_i = \frac{X_i}{1 - X_i}$ and $U_n = \frac{1}{n} \sum$

15. Let X and Y be two independent N (0, 1) random variables. Then $P(0 < X^2 + Y^2 < 4)$ equals

(A) $1 - e^{-2}$ (B) $1 - e^{-4}$

(C) $1 - e^{-1}$ (D) e^{-2}

16. Let X be a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{8}, & 0 \le x < 2, \\ \frac{x^2}{16}, & 2 \le x < 4 \\ 1, & x \ge 4 \end{cases}$$

Then $E(X)$ equals

(A) $\frac{12}{31}$ (B) $\frac{13}{12}$

(C) $\frac{31}{21}$ (D) $\frac{31}{12}$

17. Let X_1, ..., X_n be a random sample from a population with the probability density function

$$f(x) = \frac{1}{2\theta} e^{-|x|/\theta}, x \in R, \theta > 0.$$

For a suitable constant K, the critical region of the most powerful test for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ is of the form

(A) $\sum_{i=1}^{n} |X_i| > K$ (B) $\sum_{i=1}^{n} |X_i| < K$

(C) $\sum_{i=1}^{n} \frac{1}{|X_i|} < K$ (D) $\sum_{i=1}^{n} \frac{1}{|X_i|} > K$

18. Let X_1, ..., X_n, X_{n+1}, X_{n+2}, ..., X_{n+m} (n > 4, m > 4) be a random sample from $N(\mu, \sigma^2)$; $\mu \in \mathbb{R}$, $\sigma > 0$. If $\overline{X}_1$ and $\frac{1}{n} \sum$

19. Let X_1, ..., X_n (n > 1) be a random sample from a *Poisson* (θ) population, $\theta > 0$, and $T = \sum_{i=1}^{n} X_i$. Then the uniformly minimum variance unbiased estimator of θ^2 is

(A) $\dfrac{T(T-1)}{n^2}$ (B) $\dfrac{T(T-1)}{n(n-1)}$

(C) $\dfrac{T(T-1)}{n(n+1)}$ (D) $\dfrac{T^2}{n^2}$

20. Let X be a random variable whose probability mass functions $f(x \mid H_0)$ (under the null hypothesis H_0) and $f(x \mid H_1)$ (under the alternative hypothesis H_1) are given by

$X = x$	0	1	2	3
$f(x \mid H_0)$	0.4	0.3	0.2	0.1
$f(x \mid H_1)$	0.1	0.2	0.3	0.4

For testing the null hypothesis $H_0 : X \sim f(x \mid H_0)$ against the alternative hypothesis $H_1 : X \sim f(x \mid H_1)$, consider the test given by : Reject H_0 if $X > \dfrac{3}{2}$.

If α = size of the and β = power of the test, then

(A) $\alpha = 0.3$ and $\beta = 0.3$ (B) $\alpha = 0.3$ and $\beta = 0.7$

(C) $\alpha = 0.7$ and $\beta = 0.3$ (D) $\alpha = 0.7$ and $\beta = 0.7$

21. Let X_1, ..., X_n be a random sample from a $N(2\theta, \theta^2)$ population, $\theta > 0$. A consistent estimator for θ is

(A) $\dfrac{1}{n}\sum_{i=1}^{n} X_i$ (B) $\left(\dfrac{5}{n}\sum_{i=1}^{n} X_i^2\right)^{1/2}$

(C) $\dfrac{1}{5n}\sum_{i=1}^{n} X_i^2$ (D) $\left(\dfrac{1}{5n}\sum_{i=1}^{n} X_i^2\right)^{1/2}$

22. An institute purchases laptops from either vendor V_1 or vendor V_2 with equal probability. The lifetimes (in years) of laptops from vendor V_1 have a U (0,4) distribution, and the lifetimes (in years) of laptops from vendor V_2 have an Exp (1/2) distribution. If a randomly selected laptop in the institute has lifetime more than two years, then the probability that it was supplied by vendor V_2 is

(A) $\dfrac{2}{2+e}$ (B) $\dfrac{1}{1+e}$

(C) $\dfrac{1}{1+e^{-1}}$ (D) $\dfrac{2}{2+e^{-1}}$

23. Let $y(x)$ be the solution to the differential equation

$$x^4 \frac{dy}{dx} + 4x^3 y + \sin x = 0; \; y(\pi) = 1, \; x > 0$$

Then $y\left(\frac{\pi}{2}\right)$ is

(A) $\frac{10(1+\pi^4)}{\pi^4}$

(B) $\frac{12(1+\pi^4)}{\pi^4}$

(C) $\frac{14(1+\pi^4)}{\pi^4}$

(D) $\frac{16(1+\pi^4)}{\pi^4}$

24. Let $a_n = e^{-2n} \sin n$ and $b_n = e^{-n} n^2 (\sin n)^2$ for $n \geq 1$. Then

(A) $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} b_n$ does NOT converge

(B) $\sum_{n=1}^{\infty} b_n$ converges but $\sum_{n=1}^{\infty} a_n$ does NOT converge

(C) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge

(D) NEITHER $\sum_{n=1}^{\infty} a_n$ NOR $\sum_{n=1}^{\infty} b_n$ converges

25. Let

$$f(x) = \begin{cases} x \sin^2(1/x), & x \neq 0, \\ 0, & x = 0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x(\sin x)\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Then

(A) f is differentiable at 0 but g is NOT differentiable at 0

(B) g is differentiable at 0 but f is NOT differentiable at 0

(C) f and g are both differentiable at 0

(D) NEITHER f NOR g is differentiable at 0

26. Let $f : [0, 4] \to \mathbb{R}$ be a twice differentiable function. Further, let $f(0) = 1$, $f(2) = 2$ and $f(4) = 3$. Then

(A) there does NOT exist any $x_1 \in (0,2)$ such that $f'(x_1) = \frac{1}{2}$

(B) there exist $x_2 \in (0,2)$ and $x_3 \in (2,4)$ such that $f'(x_2) = f'(x_3)$

(C) $f''(x) > 0$ for all $x \in (0,4)$

(D) $f''(x) < 0$ for all $x \in (0,4)$

27. Let f (x, y) = $x^2 - 400\,xy^2$ for all $(x,y) \in \mathbb{R}^2$. Then f attains its

(A) local minimum at (0,0) but NOT at (1,1)

(B) local minimum at (1,1) but NOT at (0,0)

(C) local minimum both at (0,0) and (1,1)

(D) local minimum NEITHER at (0,0) NOR at (1,1)

28. Let y(x) be the solution to the differential equation

$$4\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 9y = 0,\ y(0) = 1,\ y'(0) = -4$$

Then y (1) equals

(A) $-\frac{1}{2}e^{-3/2}$ (B) $-\frac{3}{2}e^{-3/2}$

(C) $-\frac{5}{2}e^{-3/2}$ (D) $-\frac{7}{2}e^{-3/2}$

29. Let g : [0,2] $\to \mathbb{R}$ be defined by

$$g(x) = \int_0^x (x-t)e^t\,dt.$$

The area between the curve y = g" (x) and the x-axis over the interval [0,2] is

(A) $e^2 - 1$ (B) $2(e^2 - 1)$

(C) $4(e^2 - 1)$ (D) $8(e^2 - 1)$

30. Let P be a 3 × 3 singular matrix such that $P\vec{v} = \vec{v}$ for a nonzero vector $\vec{v}$ and

$$P\begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}2/5\\0\\-2/5\end{bmatrix}.$$

Then

(A) $P^3 = \frac{1}{5}(7P^2 - 2P)$ (B) $P^3 = \frac{1}{4}(7P^2 - 2P)$

(C) $P^3 = \frac{1}{3}(7P^2 - 2P)$ (D) $P^3 = \frac{1}{2}(7P^2 - 2P)$

SECTION – B MULTIPLE SELECT QUESTIONS (MSQ)

Q.31 to Q.40 carry one marks each.

31. For two nonzero real numbers a and b, consider the system of linear equations

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b/2 \\ a/2 \end{bmatrix}.$$

Which of the following statements is (are) TRUE ?

(A) If $a = b$, the solutions of the system lie on the line $x + y = 1/2$

(B) If $a = -b$, the solutions of the system lie on the line $y - x = 1/2$

(C) If $a \neq \pm b$, the system has no solution

(D) If $a \neq \pm b$, the system has a unique solution

32. For $n \geq 1$, let

$$a_n = \begin{cases} n2^{-n}, \text{ if n is odd} \\ -3^{-n}, \text{ if n is even} \end{cases}$$

Which of the following statements is (are) TRUE ?

(A) The sequence $\{a_n\}$ converges

(B) The sequence $\{|a_n|^{1/n}\}$ converges

(C) The series $\sum_{n=1}^{\infty} a_n$ converges

(D) The series $\sum_{n=1}^{\infty} |a_n|$ converges

33. Let $f : (0,\infty) \to \mathbb{R}$ be defined by

$$f(x) = x\left(e^{1/x^3} - 1 + \frac{1}{x^3}\right)$$

Which of the following statements is (are) TRUE ?

(A) $\lim_{x \to \infty} f(x)$ exists

(B) $\lim_{x \to \infty} xf(x)$ exists

(C) $\lim_{x \to \infty} x^2 f(x)$ exists

(D) There exists $m > 0$ such that $\lim_{x \to \infty} x^m f(x)$ does NOT exist.

34. For $x \in \mathbb{R}$, define $f(x) = \cos(\pi x) + [x^2]$ and $g(x) = \sin(\pi x)$. Which of the following statements is (are) TRUE ?

(A) $f(x)$ is continuous at $x = 2$

(B) $g(x)$ is continuous at $x = 2$

(C) $f(x) + g(x)$ is continuous at $x = 2$

(D) $f(x)\, g(x)$ is continuous at $x = 2$

35. Let E and F be two events with $0 < P(E) < 1$, $0 < P(F) < 1$ and $P(E \mid F) > P(E)$. Which of the following statements is (are) TRUE?

(A) $P(F \mid E) > P(F)$

(B) $P(E \mid F^C) > P(E)$

(C) $P(F \mid E^C) < P(F)$

(D) E and F are independent

36. Let X_1, ..., X_n $(n > 1)$ be a random sample from a $U\,(2\theta - 1, 2\theta + 1)$ population, $\theta \in \mathbb{R}$, and $Y_1 = \min\{X_1, ..., X_n\}$, $Y_n = \max\{X_1, ..., X_n\}$. Which of the following statistics is (are) maximum likelihood estimator (s) of θ ?

(A) $\frac{1}{4}\,(Y_1 + Y_n)$

(B) $\frac{1}{6}\,(2Y_1 + Y_n + 1)$

(C) $\frac{1}{8}\,(Y_1 + 3Y_n - 2)$

(D) Every statistic $T\,(X_1,....,X_n)$ satisfying $\frac{(Y_n - 1)}{2} < T\,(X_1,.....,X_n) < \frac{(Y_1 + 1)}{2}$

37. Let X_1,....,X_n be a random sample from a $N\,(0, \sigma^2)$ population, $\sigma > 0$. Which of the following testing problems has (have) the region $\left\{(x_1,....,x_n) \in R^n : \sum_{i=1}^{n} x_i^2 \geq \chi^2_{n,\alpha}\right\}$ as the most powerful critical region of level α ?

(A) $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 2$

(B) $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 4$

(C) $H_0 : \sigma^2 = 2$ against $H_1 : \sigma^2 = 1$

(D) $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 0.5$

38. Let X_1,...., X_n be a random sample from a $N\,(0, 2\theta^2)$ population, $\theta > 0$. Which of the following statements is (are) TRUE ?

(A) $(X_1,....., X_n)$ is sufficient and complete

(B) $(X_1,....., X_n)$ is sufficient but NOT complete

(C) $\sum_{i=1}^{n} X_i^2$ is sufficient and complete

(D) $\frac{1}{2n}\sum_{i=1}^{n} X_1^2$ is the uniformly minimum variance unbiased estimator for θ^2

39. Let X_1,......,X_n be a random sample from a population with the probability density function

$$f(x \mid \theta) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}, \theta > 0$$

Which of the following is (are) 100 $(1 - \alpha)$% confidence interval(s) for θ ?

(A) $\left(\frac{\chi^2_{2n,1-\alpha/2}}{2\sum_{i=1}^{n} X_i}, \frac{\chi^2_{2n,\alpha/2}}{2\sum_{i=1}^{n} X_i}\right)$

(B) $\left(0, \frac{\chi^2_{2n,\alpha}}{2\sum_{i=1}^{n} X_i}\right)$

(C) $\left(\frac{\chi^2_{2n,1-\alpha/2}}{\sum_{i=1}^{n} X_i}, \frac{\chi^2_{2n,\alpha/2}}{\sum_{i=1}^{n} X_i}\right)$

(D) $\left(\frac{2\sum_{i=1}^{n} X_i}{\chi^2_{2n,\alpha/2}}, \frac{2\sum_{i=1}^{n} X_i}{\chi^2_{2n,1-\alpha/2}}\right)$

40. The cumulative distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{10}\left(x^2 - \frac{7}{3}\right), & 2 \le x < 3 \\ 1, & x \ge 3. \end{cases}$$

Which of the following statements is (are) TRUE ?

(A) $F(x)$ is continuous everywhere

(B) $F(x)$ increases only by jumps

(C) $P(X = 2) = \frac{1}{6}$

(D) $P\left(X = \frac{5}{2} \mid 2 \le X \le 3\right) = 0$

SECTION – C NUMERICAL ANSWER TYPE (NAT)

Q.41 – Q.50 carry one mark each.

41. Let $X_1, \ldots, X_{10}$ be a random sample from a $N(3,12)$ population. Suppose $Y_1 = \frac{1}{6}\sum_{i=1}^{6} X_i$ and $Y_2 = \frac{1}{4}\sum_{i=7}^{10} X_i$. If $\frac{(Y_1 - Y_2)^2}{\alpha}$ has a χ_1^2 distribution, then the value of α is __________.

42. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{2x}{9}, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Then the upper bound of $P(|X - 2| > 1)$ using Chebyshev's inequality is ___________.

43. Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} e^{(x+y)}, & -\infty < x, y < 0, \\ 0, & \text{otherwise} \end{cases}$$

Then $P(X < Y) =$ ______________.

44. Let X and Y be continuous random variables with the joint probability density function

$$f(x,y) = \frac{1}{2\pi} e^{-\left(x^2 + y^2\right)/2}, \quad (x, y) \in \mathbb{R}^2.$$

Then $P(X > 0, Y < 0) =$ _________________.

45. Let Y be a $Bin\left(72, \frac{1}{3}\right)$ random variable. Using normal approximation to binomial distribution, an approximate value of $P(22 \le Y \le 28)$ is __________.

46. Let X be a *Bin* (2,p) random variable and Y be a *Bin* (4,p) random variable, $0 < p < 1$. If $P(X \geq 1) = \frac{5}{9}$, then $P(Y \geq 1) =$ _________.

47. Consider the linear transformation

$$T(x, y, z) = (2x + y + z, x + z, 3x + 2y + z).$$

The rank of T is ___________.

48. The value of $\lim_{n \to \infty} n \left[e^{-n} \cos(4n) + \sin\left(\frac{1}{4n}\right) \right]$ is ______________.

49. Let $f : [0, 13] \to \mathbb{R}$ be defined by $f(x) = x^{13} - e^{-x} + 5x + 6$. The minimum value of the function f on [0,13] is___________.

50. Consider a differentiable function f on [0,1] with the derivative $f'(x) = 2\sqrt{2x}$. The arc length of the curve $y = f(x)$, $0 \leq x \leq 1$, is _____________.

Q. 51 – Q. 60 carry two marks each.

51. Let m be a real number such that $m > 1$. If $\int_1^m \int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx\, dz = e - 1$, then $m =$ ____________.

52. Let $P = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$. The product of the eigen values of P^{-1} is _____________.

53. The value of the real number m in the following equation

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x^2 + y^2)\, dy\, dx = \int_{m\pi}^{\pi/2} \int_0^{\sqrt{2}} r^3\, dr\, d\theta$$ is ____________.

54. Let $a_1 = 1$ and $a_n = 2 - \frac{1}{n}$ for $n \geq 2$ Then $\sum_{n=1}^{\infty} \left(\frac{1}{a_n^2} - \frac{1}{a_{n+1}^2} \right)$ converges to ____________.

55. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with the probability density function $f(x) = \begin{cases} 4x^2 e^{-2x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}$ and let $S_n = \sum_{i=1}^{n} X_i$. Then $\lim_{n \to \infty} P\left(S_n \leq \frac{3n}{2} + \sqrt{3n}\right)$ is __________.

56. Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} \frac{cx^2}{y^3}, & 0 < x < 1, y > 1 \\ 0, & \text{otherwise} \end{cases}$$

where c is a suitable constant. Then $E(X) =$ _____________.

57. Two points are chosen at random on a line segment of length 9 cm. The probability that the distance between these two points is less than 3 cm is ___________.

58. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{X+1}{2}, & -1 < X < 1, \\ 0, & \text{Otherwise} \end{cases}$$

Then $P\left(\frac{1}{4} < X^2 < \frac{1}{2}\right) =$ ___________.

59. If X is a $U(0,1)$ random variable, then $P\left(\min(X, 1-X) \le \frac{1}{4}\right) =$ __________.

60. In a colony all families have at least one child. The probability that a randomly chosen family from this colony has exactly k children is $(0.5)^k$; $k = 1, 2, \ldots$. A child is either a male or a female with equal probability. The probability that such a family consists of at least one male child and at least one female child is _________.

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
C	A	C	B	B	C	D	A	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	C	B	C	A	D	A	C	A	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	A	D	C	B	B	D	B	A	A

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
A,B,D	A,C,D	A,B,C,D	B,D	A,C	A,B,C,D	A,B	B,C,D	A,B	C,D

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	42	43	44	45	46	47	48	49	50
5	0.5	0.5	0.25	0.53	0.80	2	0.25	5	2.167
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
4	-0.50	0.25	0.75	0.97	0.75	0.55	0.20	0.50	0.25

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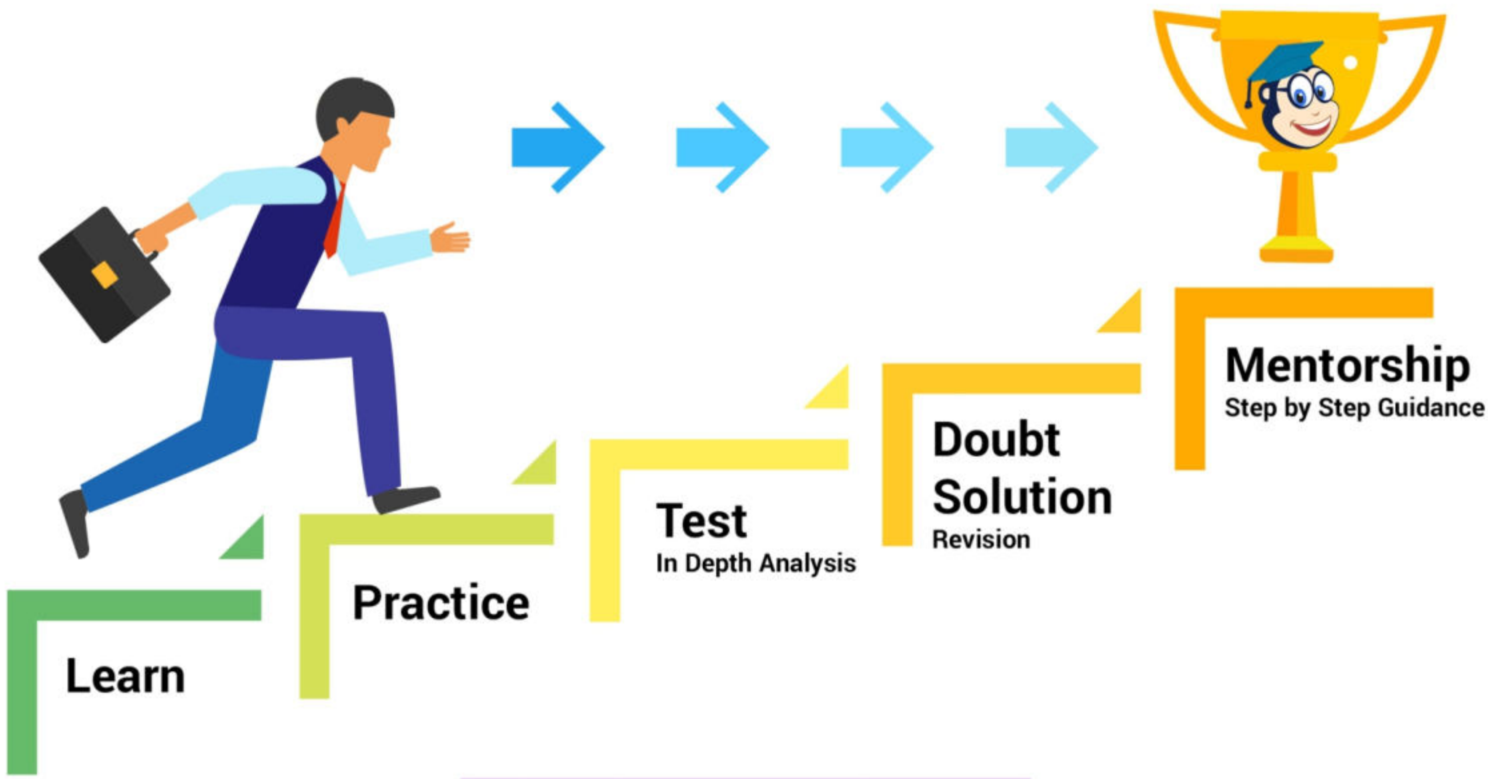

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