Sample Stokes' and Divergence Theorem questions

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These are taken from old 103 finals from Clark Bray. Full solutions are available on his web site `http://www.math.duke.edu/~cbray/` (these are from his Summer 2005 and Fall 2005-06 classes). *Note:* Stokes' Theorem and the Divergence Theorem are not necessarily applicable to all of these.

1. Compute the flux of $\vec{F}$ through S, where:
   (a) $\vec{F} = \vec{\nabla} \times (y + ze^{xy}, -x + ze^z, x^2y^2)$, S is the part of the unit sphere above the xy-plane, oriented away from the origin (answer: -2π);
   (b) $\vec{F} = (x^2, x^2z + e^z, xy^2)$, S is the surface of the box $[0, 2] \times [0, 1] \times [0, 3]$, oriented outwards (answer: 12);
   (c) same as (b) but S is missing the bottom $[0, 2] \times [0, 1] \times \{0\}$ (answer: $38/3$, I think; how would you use (b) to calculate this?).

2. Let B be the ball in $\mathbb{R}^3$ of radius 5, centered at the origin. Compute
$$\iiint_B (\vec{\nabla} \cdot \vec{F})\, dV$$
where $\vec{F} = (-yze^{x^2y^3 \sin z}, 3xze^{x^2y^3 \sin z}, -2xye^{x^2y^3 \sin z})$. (Answer: 0.)

3. (a) Compute the flux $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = (e^y, -e^x, 0)$, and S is the portion above the xy-plane of the surface defined by $x^2 + 5y^2 - (z - 3)^2 = 16$, oriented outward (at the point $(0, 0, 7)$, the unit normal vector is $\hat{k}$). (Answer: 0, but *be careful*: S isn't a closed surface.)
   (b) Compute the flux $\iint_S (\vec{\nabla} \times \vec{G}) \cdot d\vec{S}$, where $\vec{G} = (0, 0, e^{xyz})$. (Answer: 0.)

4. Compute the flux $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = (2y, z, -x)$, and S is the part of the plane $x + y + z = 4$ that is above the rectangle in the xy-plane with vertices at $(0, 0, 0)$, $(2, 0, 0)$, $(0, 1, 0)$, and $(2, 1, 0)$, oriented upwards. (Answer: 5. You can't use Stokes for this one!)