5 Previous Year Papers with Answer Key (2016 to 2020)

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Mathematics (MA)

Previous Year Solved Paper 2020

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ),** wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.

NOTATION

N set of all natural numbers 1, 2, 3,.......

R set of all real numbers

$M_{m \times n}(R)$ real vector space of all matrices of size m × n with entries in R

ϕ empty set

X/Y set of all elements from the set X which are not in the set Y

Z_n group of all congruence classes of integers modulo n

$\hat{i}, \hat{j}, \hat{k}$ unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system, respectively

S_n group of all permutations of the set {1, 2, 3,.....,n}.

ln logarithm to the base e

log logarithm to the base 10

∇ $\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

det(M) determinant of a square matrix M

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1. The radius of convergence of the power series $\sum_{n=1}^{\infty}\left(\frac{n+2}{n}\right)^{n^2} x^n$ is

(A) e^2 (B) $\frac{1}{\sqrt{e}}$

(C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$

2. Which of the following is FALSE?

(A) $\lim_{x\to\infty}\frac{x}{e^x}=0$ (B) $\lim_{x\to 0^+}\frac{1}{xe^{1/x}}=0$

(C) $\lim_{x\to 0^+}\frac{\sin x}{1+2x}=0$ (D) $\lim_{x\to 0^+}\frac{\cos x}{1+2x}=0$

3. Let $f(x) = 2x^3 - 9x^2 + 7$. Which of the following is true?

(A) f is one-one in the interval [–1, 1]

(B) f is one-one in the interval [2, 4]

(C) f is NOT one-one in the interval [–4, 0]

(D) f is NOT one-one in the interval [0, 4]

4. If $u = x^3$ and $v = y^2$ transform the differential equation $3x^5dx - y(y^2 - x^3)dy = 0$ to $\frac{dv}{du}=\frac{\alpha u}{2(u-v)}$, then α is

(A) 4 (B) 2

(C) –2 (D) –4

5. Consider the following group under matrix multiplication:

$$H=\left\{\begin{bmatrix}1 & p & q\\ 0 & 1 & r\\ 0 & 0 & 1\end{bmatrix} : p,q,r\in\mathbb{R}\right\}$$

Then the center of the group is isomorphic to

(A) $(\mathbb{R}\setminus\{0\}, \times)$ (B) $(\mathbb{R}, +)$

(C) $(\mathbb{R}^2, +)$ (D) $(\mathbb{R}, +) \times (\mathbb{R}\setminus\{0\}, \times)$

6. If the equation of the tangent plane to the surface $z = 16 - x^2 - y^2$ at the point P(1,3,6) is $ax + by + cz + d = 0$, then the value of $|d|$ is

(A) 16 (B) 26

(C) 36 (D) 46

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(x, y) = (-x, y)$. Then

(A) $T^{2k} = T$ for all $k \geq 1$

(B) $T^{2k+1} = -T$ for all $k \geq 1$

(C) The range of T^2 is a proper subspace of the range of T

(D) The range of T^2 is equal to the range of T

8. Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $f(x,y) = g(y) + xg'(y)$, then

(A) $\frac{\partial f}{\partial x} + y\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial y}$

(B) $\frac{\partial f}{\partial y} + y\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x}$

(C) $\frac{\partial f}{\partial x} + x\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial y}$

(D) $\frac{\partial f}{\partial y} + x\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x}$

9. Let $S_n = 1 + \frac{(-1)^n}{n}, n \in \mathbb{N}$. Then the sequence $\{s_n\}$ is

(A) monotonically increasing and is convergent to 1

(B) monotonically decreasing and is convergent to 1

(C) neither monotonically increasing nor monotonically decreasing but is convergent to 1

(D) divergent

10. If the directional derivative of the function $z = y^2e^{2x}$ at (2, –1) along the unit vector $\vec{b} = \alpha\hat{i} + \beta\hat{j}$ is zero, then $|\alpha + \beta|$ equals

(A) $\frac{1}{2\sqrt{2}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\sqrt{2}$

(D) $2\sqrt{2}$

11. Let $F = \{\omega \in \mathbb{C} : \omega^{2020} = 1\}$. Consider the groups $G = \left\{\begin{pmatrix} \omega & z \\ 0 & 1 \end{pmatrix} : \omega \in F, z \in \mathbb{C}\right\}$ and $H = \left\{\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} : z \in \mathbb{C}\right\}$ under matrix multiplication. Then the number of cosets of H in G is

(A) 1010

(B) 2019

(C) 2020

(D) infinite

12. Define $s_1 = \alpha > 0$ and $s_{n+1} = \sqrt{\frac{1+s_n^2}{1+\alpha}}, n \geq 1$. Which of the following is true?

(A) If $s_n^2 < \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim_{n\to\infty} s_n = \frac{1}{\sqrt{\alpha}}$

(B) If $s_n^2 < \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim_{n\to\infty} s_n = \frac{1}{\alpha}$

(C) If $s_n^2 > \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim_{n\to\infty} s_n = \frac{1}{\sqrt{\alpha}}$

(D) If $s_n^2 > \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim_{n\to\infty} s_n = \frac{1}{\alpha}$

13. Let $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ be the circle group under multiplication and $i = \sqrt{-1}$. Then the set $\{\theta \in \mathbb{R} : \langle e^{i2\pi\theta} \rangle \text{is infinite}\}$ is

(A) empty (B) non-empty and finite

(C) countably infinite (D) uncountable

14. Let M be a 4 × 3 real matrix and let $\{e_1, e_2, e_3\}$ be the standard basis of $\mathbb{R}^3$. Which of the following is true?

(A) If rank (M) = 1, then $\{Me_1, Me_2\}$ is a linearly independent set

(B) If rank (M) = 2, then $\{Me_1, Me_2\}$ is a linearly independent set

(C) If rank (M) = 2, then $\{Me_1, Me_3\}$ is a linearly independent set

(D) If rank (M) = 3, then $\{Me_1, Me_3\}$ is a linearly independent set

15. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $x, y, z \in \mathbb{R}$.. Which of the following is FALSE?

(A) $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$ (B) $\nabla \cdot (\vec{a} \times \vec{r}) = 0$

(C) $\nabla \times (\vec{a} \times \vec{r}) = \vec{a}$ (D) $\nabla \cdot ((\vec{a} \cdot \vec{r})\vec{r}) = 4(\vec{a} \cdot \vec{r})$

16. Consider the differential equation $L[y] = (y - y^2)dx + xdy = 0$. The function f(x, y) is said to be an integrating factor of the equation if f(x, y)L[y] = 0 becomes exact.

If $f(x, y) = \frac{1}{x^2y^2}$, then

(A) f is an integrating factor and $y = 1 - kxy$, $k \in \mathbb{R}$ is NOT its general solution

(B) f is an integrating factor and $y = -1 + kxy$, $k \in \mathbb{R}$ is its general solution

(C) f is an integrating factor and $y = -1 + kxy$, $k \in \mathbb{R}$ is NOT its general solution

(D) f is NOT an integrating factor and $y = 1 + kxy$, $k \in \mathbb{R}$ is its general solution

17. Let $\{a_n\}$ be a sequence of positive real numbers. Suppose that $l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Which of the following is true?

(A) If $l = 1$, then $\lim_{n\to\infty} a_n = 1$ (B) If $l = 1$, then $\lim_{n\to\infty} a_n = 0$

(C) If $l < 1$, then $\lim_{n\to\infty} a_n = 1$ (D) If $l < 1$, then $\lim_{n\to\infty} a_n = 0$

18. Let $D = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \le 1\}$ and $f : D \to \mathbb{R}$ be a non-constant continuous function.

Which of the following is TRUE?

(A) The range of f is unbounded

(B) The range of f is a union of open intervals

(C) The range of f is a closed interval

(D) The range of f is a union of at least two disjoint closed intervals

19. A solution of the differential equation $2x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-y=0,\ x>0$ that passes through the point (1,1) is

(A) $y=\frac{1}{x}$ (B) $y=\frac{1}{x^2}$

(C) $y=\frac{1}{\sqrt{x}}$ (D) $y=\frac{1}{x^{3/2}}$

20. Let $f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$, A point at which the gradient of the function f is equal to zero is

(A) (–1, 1, –1) (B) (–1, –1, –1)

(C) (–1, 1, 1) (D) (1, –1, 1)

21. Let $f : [0,1] \to \mathbb{R}$ be a continuous function such that $f\left(\frac{1}{2}\right)=-\frac{1}{2}$ and

$$|f(x) - f(y) - (x - y)| \le \sin(|x - y|^2)$$

for all $x, y \in [0,1]$. Then $\int_0^1 f(x)\,dx$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

22. The area bounded by the curves $x^2 + y^2 = 2x$ and $x^2 + y^2 = 4x$, and the straight lines $y = x$ and $y = 0$ is

(A) $3\left(\frac{\pi}{2}+\frac{1}{4}\right)$ (B) $3\left(\frac{\pi}{4}+\frac{1}{2}\right)$

(C) $2\left(\frac{\pi}{4}+\frac{1}{3}\right)$ (D) $2\left(\frac{\pi}{3}+\frac{1}{4}\right)$

23. The value of the triple integral $\iiint_V (x^2y+1)$ dxdydz, where V is the region given by $x^2 + y^2 \le 1$, $0 \le z \le 2$ is

(A) π (B) 2π

(C) 3π (D) 4π

24. Let S be the part of the cone $z^2 = x^2 + y^2$ between the planes $z = 0$ and $z = 1$. Then the value of the surface integral $\iint_S (x^2+y^2)\,dS$ is

(A) π (B) $\frac{\pi}{\sqrt{2}}$

(C) $\frac{\pi}{\sqrt{3}}$ (D) $\frac{\pi}{2}$

25. Let M be an n × n (n ≥ 2) non-zero real matrix with $M^2 = 0$ and $\alpha \in \mathbb{R}\backslash\{0\}$. Then

(A) α is the only eigenvalue of $(M + \alpha I)$ and $(M - \alpha I)$

(B) α is the only eigenvalue of $(M + \alpha I)$ and $(\alpha I - M)$

(C) $-\alpha$ is the only eigenvalue of $(M + \alpha I)$ and $(M - \alpha I)$

(D) $-\alpha$ is the only eigenvalue of $(M + \alpha I)$ and $(\alpha I - M)$

26. Let $a \in \mathbb{R}$. If $f(x) = \begin{cases} (x+a)^2, & x \le 0 \\ (x+a)^3, & x > 0, \end{cases}$

then

(A) $\frac{d^2f}{dx^2}$ does not exist at x = 0 for any value of a

(B) $\frac{d^2f}{dx^2}$ exists at x = 0 for exactly one value of a

(C) $\frac{d^2f}{dx^2}$ exists at x = 0 for exactly two values of a

(D) $\frac{d^2f}{dx^2}$ exists at x = 0 for infinitely many values of a

27. Suppose that S is the sum of a convergent series $\sum_{n=1}^{\infty} a_n$. Define $t_n = a_n + a_{n+1} + a_{n+2}$. Then the series $\sum_{n=1}^{\infty} t_n$

(A) diverges

(B) converges to $3S - a_1 - a_2$

(C) converges to $3S - a_1 - 2a_2$

(D) converges to $3S - 2a_1 - a_2$

28. Let M be a real 6 × 6 matrix. Let 2 and –1 be two eigenvalues of M. If $M^5 = aI + bM$, where a, b $\in \mathbb{R}$, then

(A) a = 10, b = 11

(B) a = –11, b = 10

(C) a = –10, b = 11

(D) a = 10, b = –11

29. Let S be the surface of the portion of the sphere with centre at the origin and radius 4, above the xy-plane. Let $\vec{F} = y\hat{i} - x\hat{j} + yx^3\hat{k}$. If $\hat{n}$ is the unit outward normal to S, then

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$$

equals

(A) -32π

(B) -16π

(C) 16π

(D) 32π

30. Let $f(x,y)=\begin{cases} x^2\sin\frac{1}{x}+y^2\sin\frac{1}{y}, & xy\neq 0 \\ x^2\sin\frac{1}{x}, & x\neq 0, y=0 \\ y^2\sin\frac{1}{y}, & y\neq 0, x=0 \\ 0, & x=y=0 \end{cases}$

Which of the following is true at (0, 0)?

(A) f is not continuous

(B) $\frac{\partial f}{\partial x}$ is continuous but $\frac{\partial f}{\partial y}$ is not continuous

(C) f is not differentiable

(D) f is differentiable but both $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are not continuous

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31. Consider the following system of linear equations

$x + y + 5z = 3,\ x + 2y + mz = 5$ and $x + 2y + 4z = k.$

The system is consistent if

(A) $m \neq 4$ (B) $k \neq 5$

(C) $m = 4$ (D) $k = 5$

32. Let f be a real valued function of a real variable, such that $\left|f^{(n)}(0)\right| \leq K$ for all $n \in \mathbb{N}$, where K > 0, Which of the following is/are true?

(A) $\left|\frac{f^{(n)}(0)}{n!}\right|^{\frac{1}{n}} \to 0$ as $n \to \infty$

(B) $\left|\frac{f^{(n)}(0)}{n!}\right|^{\frac{1}{n}} \to \infty$ as $n \to \infty$

(C) $f^{(n)}(x)$ exists for all $x \in \mathbb{R}$ and for all $n \in \mathbb{N}$

(D) The series $\sum_{n=1}^{\infty}\frac{f^{(n)}(0)}{(n-1)!}$ is absolutely convergent

33. Let V be a non-zero vector space over a field F. Let $S \subset V$ be a non-empty set. Consider the following properties of S:

(I) For any vector space W over F, any map $f : S \to W$ extends to a linear map from V to W.

(II) For any vector space W over F and any two linear maps $f, g : V \to W$ satisfying $f(s) = g(s)$ for all $s \in S$, we have $f(v) = g(v)$ for $v \in V$.

(III) S is linearly independent.

(IV) The span of S is V.

Which of the following statement(s) is/are true?

(A) (I) implies (IV)
(B) (I) implies (III)
(C) (II) implies (III)
(D) (II) implies (IV)

34. Let G be a group with identity e. Let H be an abelian non-trivial proper subgroup of G with the property that $H \cap gHg^{-1} = \{e\}$ for all $g \notin H$.

If $K = \{g \in G : gh = hg \text{ for all } h \in H\}$, then

(A) K is a proper subgroup of H
(B) H is a proper subgroup of K
(C) K = H
(D) there exists no abelian subgroup $L \subseteq G$ such that K is a proper subgroup of L

35. Let $L[y] = x^2\dfrac{d^2y}{dx^2} + px\dfrac{dy}{dx} + qy$, where p, q are real constants. Let $y_1(x)$ and $y_2(x)$ be two solutions of $L[y] = 0$, $x > 0$, that satisfy $y_1(x_0) = 1$, $y'_1(x_0) = 0$, $y_2(x_0) = 0$ and $y'_2(x_0) = 1$ for some $x_0 > 0$. Then,

(A) $y_1(x)$ is not a constant multiple of $y_2(x)$
(B) $y_1(x)$ is a constant multiple of $y_2(x)$
(C) 1, ln x are solutions of $L[y] = 0$ when $p = 1$, $q = 0$
(D) x, ln x are solutions of $L[y] = 0$ when $p + q \neq 0$

36. Let $a = \lim\limits_{x\to\infty}\left(\dfrac{1}{n^2} + \dfrac{2}{n^2} + \ldots + \dfrac{(n-1)}{n^2}\right)$ and $b = \lim\limits_{n\to\infty}\left(\dfrac{1}{n+1} + \dfrac{1}{n+2} + \ldots + \dfrac{1}{n+n}\right)$.

Which of the following is/are true?

(A) a > b
(B) a < b
(C) $ab = \ln\sqrt{2}$
(D) $\dfrac{a}{b} = \ln\sqrt{2}$

37. Let $a, b \in \mathbb{R}$ and a < b. Which of the following statement(s) is/are true?

(A) There exists a continuous function $f : [a,b] \to (a, b)$ such that f is one-one
(B) There exists a continuous function $f : [a,b] \to (a, b)$ such that f is onto
(C) There exists a continuous function $f : (a,b) \to [a, b]$ such that f is one-one
(D) There exists a continuous function $f : (a,b) \to [a, b]$ such that f is onto

38. Let S be that part of the surface of the paraboloid $z = 16 - x^2 - y^2$ which is above the plane z = 0 and D be its projection on the xy-plane. Then the area of S equals

(A) $\iint\limits_D \sqrt{1+4(x^2+y^2)}\,dxdy$
(B) $\iint\limits_D \sqrt{1+2(x^2+y^2)}\,dxdy$
(C) $\int_0^{2\pi}\int_0^4 \sqrt{1+4r^2}\,dr\,d\theta$
(D) $\int_0^{2\pi}\int_0^4 \sqrt{1+4r^2}\,r\,dr\,d\theta$

39. Let a, b, c $\in \mathbb{R}$ such that a < b < c. Which of the following is/are true for any continuous function $f : \mathbb{R} \to \mathbb{R}$ satisfying f(a) = b, f(b) = c and f(c) = a?

(A) There exists $\alpha \in (a,c)$ such that $f(\alpha) = \alpha$

(B) There exists $\beta \in (a,b)$ such that $f(\beta) = \beta$

(C) There exists $\gamma \in (a,b)$ such that $(f \circ f)(\gamma) = \gamma$

(D) There exists $\delta \in (a,c)$ such that $(f \circ f \circ f)(\delta) = \delta$

40. If $s_n \frac{(-1)^n}{2^n+3}$ and $t_n = \frac{(-1)}{4n-1}$, n = 0, 1, 2, ..., then

(A) $\sum_{n=0}^{\infty} s_n$ is absolutely convergent (B) $\sum_{n=0}^{\infty} t_n$ is absolutely convergent

(C) $\sum_{n=0}^{\infty} s_n$ is conditionally convergent (D) $\sum_{n=0}^{\infty} t_n$ is conditionally convergent

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41. Let $f(x,y) = e^x \sin y$, $x = t^3 + 1$ and $y = t^4 + t$. Then $\frac{df}{dt}$ at t = 0 is ______. (rounded off to two decimal places)

42. Consider the differential equation $\frac{dy}{dx} + 10y = f(x), x > 0,$ where f(x) is a continuous function such that $\lim_{x\to\infty} f(x) = 1$. Then the value of $\lim_{x\to\infty} y(x)$ is ______.

43. Let $f : \mathbb{R} \to \mathbb{R}$ be such that f, f', f'' are continuous functions with f > 0, f' > 0 and f'' > 0. Then $\lim_{x\to-\infty} \frac{f(x)+f'(x)}{2}$ is ________.

44. If $\int_0^1 \int_{2y}^2 e^{x^2} dxdy = k(e^4 - 1)$, then k equals ________.

45. Consider the real vector space $P_{2020} = \left\{ \sum_{i=0}^{n} a_i x^i : a_i \in \mathbb{R} \text{ and } 0 \le n \le 2020 \right\}$. Let W be the subspace given by $W = \left\{ \sum_{i=0}^{n} a_i x^i \in P_{2020} : a_i = 0 \text{ for all odd } i \right\}$. Then, the dimension of W is _______.

46. Let f(x, y) = 0 be a solution of the homogeneous differential equation

$$(2x + 5y)dx - (x + 3y)dy = 0.$$

If $f(x + \alpha, y - 3) = 0$ is a solution of the differential equation

$$(2x + 5y - 1)dx + (2 - x - 3y)dy = 0,$$

then the value of α is ________.

47. Let $S=\left\{\frac{1}{n}:n\in\mathbb{N}\right\}$ and $f:S\to\mathbb{R}$ be defined by $f(x)=\frac{1}{x}$.

Then $\max\left\{\delta:\left|x-\frac{1}{3}\right|<\delta\Rightarrow\left|f(x)-f\left(\frac{1}{3}\right)\right|<1\right\}$ is ________. (rounded off to two decimal places)

48. Let $\phi : S_3 \to S^1$ be a non-trivial non-injective group homomorphism. Then, the number of elements in the kernel of ϕ is ________.

49. Let $x_n = n^{1/n}$ and $y_n = e^{1-x_n}, n \in \mathbb{N}$. Then the value of $\lim_{n\to\infty} y_n$ is _______.

50. Let $\vec{F}=x\hat{i}+y\hat{j}+z\hat{k}$ and S be the sphere given by $(x-2)^2+(y-2)^2+(z-2)^2=4$. If $\hat{n}$ is the unit outward normal to S, then $\frac{1}{\pi}\iint_S \vec{F}\cdot\hat{n}\,dS$ is _______ .

51. Let C be the boundary of the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) oriented in the counter clockwise sense. Then, the value of the line integral $\oint_C x^2y^2dx+(x^2-y^2)dy$ is ___________. (rounded off to two decimal places)

52. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f'(x) = f(x)$ for all x. Suppose that $f(\alpha x)$ and $f(\beta x)$ are two non-zero solutions of the differential equation $4\frac{d^2y}{dx^2}-p\frac{dy}{dx}+3y=0$ satisfying $f(\alpha x)\, f(\beta x) = f(2x)$ and $f(\alpha x)\, f(-\beta x) = f(x)$. Then, the value of p is _______.

53. If $x^2 + xy^2 = c$, where $c \in \mathbb{R}$, is the general solution of the exact differential equation $M(x,y)\, dx + 2xy\, dy = 0$, then $M(1, 1)$ is _______.

54. Let $T : \mathbb{R}^7 \to \mathbb{R}^7$ be a linear transformation with Nullity(T) = 2. Then, the minimum possible value for Rank(T^2) is ________.

55. Let $f(x) = \sqrt{x}+\alpha x,\ x>0$ and $g(x) = a_0 + a_1(x-1) + a_2(x-1)^2$ be the sum of the first three terms of the Taylor series of f(x) around x = 1. If g(3) = 3, then α is _______.

56. The minimum value of the function $f(x, y) = x^2 + xy + y^2 - 3x - 6y + 11$ is _______.

57. Let $M = \begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$. Then, the value of $\det((8I - M)^3)$ is _______.

58. Consider the expansion of the function $f(x) = \dfrac{3}{(1-x)(1+2x)}$ in powers of x, that is valid in $|x| < \dfrac{1}{2}$. Then the coefficient of x^4 is _________.

59. The sum of the series $\dfrac{1}{2(2^2-1)} + \dfrac{1}{3(3^2-1)} + \dfrac{1}{4(4^2-1)}$ + ... is __________.

60. Suppose that G is a group of order 57 which is NOT cyclic. If G contains a unique subgroup H of order 19, then for any $g \notin H$, o(g) is ________.

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
D	D	D	D	B	MTA	D	C	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	A	D	D	C	C	D	C	A	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	B	B	B	B	A	D	A	A	D

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
A,D	A,C,D	B,D	C,D	A,C	A,C	A,C,D	A,D	A,C,D	A,D

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
2.70-2.72	0.1	0-∞	0.25	1011	7	0.08-0.09	3	1	32
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
0.65-0.67	8	3	3	0.5	2	-216	33	0.25	3

Mathematics (MA)

Previous Year Solved Paper 2019

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ),** wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.

NOTATION

$\mathbb{N}$	set of all natural numbers 1, 2, 3,.......
$\mathbb{R}$	set of all real numbers
$M_{m \times n}(\mathbb{R})$	real vector space of all matrices of size m × n with entries in $\mathbb{R}$
ϕ	empty set
X/Y	set of all elements from the set X which are not in the set Y
$\mathbb{Z}_n$	group of all congruence classes of integers modulo n
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system, respectively
S_n	group of all permutations of the set {1, 2, 3,.....,n}.
ln	logarithm to the base e
log	logarithm to the base 10
∇	$\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$
det(M)	determinant of a square matrix M

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 – Q.10 carry one mark each.

1. Let $a_1 = b_1 = 0$, and for each $n \geq 2$, let a_n and b_n be real numbers given by

$$a_n = \sum_{m=2}^{n} \frac{(-1)^m m}{(\log(m))^m} \text{ and } b_n = \sum_{m=2}^{n} \frac{1}{(\log(m))^m}.$$

Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?

(A) Both $\{a_n\}$ and $\{b_n\}$ are divergent

(B) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent

(C) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent

(D) Both $\{a_n\}$ and $\{b_n\}$ are convergent

2. Let $T \in M_{m \times n}(\mathbb{R})$. Let V be the subspace of $M_{n \times p}(\mathbb{R})$ defined by

$V = \{X \in M_{n \times p}(\mathbb{R}) : TX = 0\}$.

Then the dimension of V is

(A) pn – rank (T)

(B) mn – p rank (T)

(C) p(m – rank (T))

(D) p(n – rank (T))

3. Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by

$f(x, y, z) = g(x^2 + y^2 - 2z^2)$.

Then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to

(A) $4(x^2 + y^2 - 4z^2)\, g''(x^2 + y^2 - 2z^2)$

(B) $4(x^2 + y^2 + 4z^2)\, g''(x^2 + y^2 - 2z^2)$

(C) $4(x^2 + y^2 - 2z^2)\, g''(x^2 + y^2 - 2z^2)$

(D) $4(x^2 + y^2 + 4z^2)\, g''(x^2 + y^2 - 2z^2) + 8g'(x^2 + y^2 - 2z^2)$

4. Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of positive real numbers such that $na_n < b_n < n^2 a_n$ for all $n \geq 2$. If the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is 4, then the power series $\sum_{n=0}^{\infty} b_n x^n$

(A) converges for all x with $|x| < 2$

(B) converges for all x with $|x| > 2$

(C) does not converge for any x with $|x| > 2$

(D) does not converge for any x with $|x| < 2$

5. Let S be the set of all limit points of the set $\left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$. Let $\mathbb{Q}_+$ be the set of all positive rational numbers. Then

(A) $\mathbb{Q}_+ \subseteq S$

(B) $S \subseteq \mathbb{Q}_+$

(C) $S \cap (\mathbb{R} \setminus \mathbb{Q}_+) \neq \phi$

(D) $S \cap \mathbb{Q}_+ \neq \phi$

6. If $x^h y^k$ is an integrating factor of the differential equation

$$y(1 + xy)\,dx + x\,(1 - xy)\,dy = 0$$

then the ordered pair (h, k) is equal to-

(A) $(-2, -2)$ (B) $(-2, -1)$

(C) $(-1, -2)$ (D) $(-1, -1)$

7. If $y(x) = \lambda e^{2x} + e^{\beta x}$, $\beta \neq 2$, is a solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

satisfying $\frac{dy}{dx}(0) = 5$, then y(0) is equal to :

(A) 1 (B) 4

(C) 5 (D) 9

8. The equation of the tangent plane to the surface $x^2 z + \sqrt{8 - x^2 - y^4} = 6$ at the point (2, 0, 1) is :

(A) $2x + z = 5$ (B) $3x + 4z = 10$

(C) $3x - z = 10$ (D) $7x - 4z = 10$

9. The value of the integral $\int_{y=0}^{1}\int_{y=0}^{1-y^2} y \sin\left(\pi(1-x)^2\right)dxdy$ is :

(A) $\frac{1}{2\pi}$ (B) 2π

(C) $\frac{\pi}{2}$ (D) $\frac{2}{\pi}$

10. The area of the surface generated by rotating the curve $x = y^3$, $0 \le y \le 1$, about the y-axis, is

(A) $\frac{\pi}{27}10^{3/2}$ (B) $\frac{4\pi}{3}\left(10^{3/2} - 1\right)$

(C) $\frac{\pi}{27}\left(10^{3/2} - 1\right)$ (D) $\frac{4\pi}{3}10^{3/2}$

Q.11-Q.30 carry two marks each.

11. Let H and K be subgroups of Z_{144}. If the order of H is 24 and the order of K is 36, then the order of the subgroup $H \cap K$ is-

(A) 3 (B) 4

(C) 6 (D) 12

12. Let P be a 4 × 4 matrix with entries from the set of rational numbers. If $\sqrt{2} + i$, with $i = \sqrt{-1}$, is a root of the characteristic polynomial of P and I is the 4 × 4 identity matrix, then

(A) $P^4 = 4P^2 + 9I$ (B) $P^4 = 4P^2 - 9I$

(C) $P^4 = 2P^2 - 9I$ (D) $P^4 = 2P^2 + 9I$

13. The set $\left\{\frac{x}{1+x}: -1 < x < 1\right\}$, as a subset of R, is

(A) connected and compact

(B) connected but not compact

(C) not connected but compact

(D) neither connected nor compact

14. The set $\left\{\frac{1}{m}+\frac{1}{n}: m, n \in N\right\} \cup \{0\}$, as a subset of R, is

(A) compact and open

(B) compact but not open

(C) not compact but open

(D) neither compact nor open

15. For $-1 < x < 1$, the sum of the power series $1 + \sum_{n=2}^{\infty}(-1)^{n-1} n^2 x^{n-1}$ is

(A) $\frac{1-x}{(1+x)^3}$

(B) $\frac{1+x^2}{(1+x)^4}$

(C) $\frac{1-x}{(1+x)^2}$

(D) $\frac{1+x^2}{(1+x)^3}$

16. Let $f(x) = (\ln x)^2$, $x > 0$. Then

(A) $\lim_{x\to\infty} \frac{f(x)}{x}$ does not exist

(B) $\lim_{x\to\infty} f'(x) = 2$

(C) $\lim_{x\to\infty} (f(x+1) - f(x)) = 0$

(D) $\lim_{x\to\infty} (f(x+1) - f(x))$ does not exist

17. Let $f : R \to R$ be a differentiable function such that $f'(x) > f(x)$ for all $x \in R$, and $f(0) = 1$. Then $f(1)$ lies in the interval

(A) $(0, e^{-1})$

(B) $(e^{-1}, \sqrt{e})$

(C) $(\sqrt{e}, e)$

(D) (e, ∞)

18. For which one of the following value of k, the equation

$2x^3 + 3x^2 - 12x - k = 0$

has three distinct real roots ?

(A) 16

(B) 20

(C) 26

(D) 31

19. Which one of the following series is divergent ?

(A) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n} \log n$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$

20. Let S be the family of orthogonal trajectories of the family of curves $2x^2 + y^2 = k$, for $k \in R$ and $k > 0$.

If $c \in S$ and c passes through the point (1, 2), then c also passes through

(A) $\left(4, -\sqrt{2}\right)$ (B) (2, –4)

(C) $\left(2, 2\sqrt{2}\right)$ (D) $\left(4, 2\sqrt{2}\right)$

21. Let x, $x + e^x$ and $1 + x + e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If y(x) is the solutions of the same equation satisfying y(0) = 3 and y'(0) = 4, then y(1) is equal to-

(A) e + 1 (B) 2e + 3

(C) 3e + 2 (D) 3e + 1

22. The function $f(x, y) = x^3 + 2xy + y^3$ has a saddle point at

(A) (0, 0) (B) $\left(-\frac{2}{3}, -\frac{2}{3}\right)$

(C) $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ (D) (–1, –1)

23. The area of the part of the surface of the paraboloid $x^2 + y^2 + z = 8$ lying inside the cylinder $x^2 + y^2 = 4$ is-

(A) $\frac{\pi}{2}\left(17^{3/2} - 1\right)$ (B) $\pi\left(17^{3/2} - 1\right)$

(C) $\frac{\pi}{6}\left(17^{3/2} - 1\right)$ (D) $\frac{\pi}{3}\left(17^{3/2} - 1\right)$

24. Let C be the circle $(x - 1)^2 + y^2 = 1$, oriented counter clockwise. Then the value of the line integral

$\oint_c -\frac{4}{3}xy^3dx + x^4dy$ is

(A) 6π (B) 8π

(C) 12π (D) 14π

25. Let $\vec{F}(x, y, z) = 2y\hat{i} + x^2\hat{j} + xy\hat{k}$ and let c be the curve of intersection of the plane x + y + z = 1 and the cylinder $x^2 + y^2 = 1$. Then the value of $\left|\oint_c \vec{F}.d\vec{r}\right|$ is :

(A) π (B) $\frac{3\pi}{2}$

(C) 2π (D) 3π

26. The tangent line to the curve of intersection of the surface $x^2 + y^2 - z = 0$ and the plane $x + z = 3$ at the point (1, 1, 2) passes through

(A) (–1, –2, 4) (B) (–1, 4, 4)

(C) (3, 4, 4) (D) (–1, 4, 0)

27. The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$?

(A) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

28. Let $\{a_n\}$ be a sequence of positive real numbers. The series $\sum_{n=1}^{\infty} a_n$ converges if the series

(A) $\sum_{n=1}^{\infty} a_n^2$ converges (B) $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ converges

(C) $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges (D) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

29. For $\beta \in R$, define

$$f(x, y) = \begin{cases} \dfrac{x^2 |x|^\beta y}{x^4 + y^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Then, at (0, 0) the function f is-

(A) continuous for $\beta = 0$ (B) continuous for $\beta > 0$

(C) not differentiable for any β (D) continuous for $\beta < 0$

30. Let $\{a_n\}$ be a sequence of positive real numbers such that

$$a_1 = 1,\ a_{n+1}^2 - 2a_n a_{n+1} - a_n = 0 \text{ for all } n \geq 1.$$

Then the sum of the series $\sum_{n=1}^{\infty} \frac{a_n}{3^n}$ lies in the interval

(A) (1, 2] (B) (2, 3]

(C) (3, 4] (D) (4, 5]

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

31. Let G be a noncyclic group of order 4. Consider the statements I and II.

I. There is NO injective (one-one) homomorphism from G to Z_8

II. There is NO surjective (onto) homomorphism from Z_8 to G

Then

(A) I is true
(B) I is false
(C) II is true
(D) II is false

32. Let G be a nonabelian group, $y \in G$, and let the maps f, g, h from G to itself be defined by $f(x) = yxy^{-1}$, $g(x) = x^{-1}$ and $h = gog$. Then

(A) g and h are homomorphisms and f is not a homomorphism
(B) h is a homomorphism and g is not a homomorphism
(C) f is a homomorphism and g is not a homomorphism
(D) f, g and h are homomorphisms

33. Let S and T be linear transformations from a finite dimensional vector space V to itself such that $S(T(v)) = 0$ for all $v \in V$. Then

(A) rank(T) $\geq$ nullity (S)
(B) rank(S) $\geq$ nullity (T)
(C) rank(T) $\leq$ nullity (S)
(D) rank(s) $\leq$ nullity (T)

34. Let $\vec{F}$ and $\vec{G}$ be differentiable vector fields and let g be a differentiable scalar function. Then

(A) $\nabla.\left(\vec{F}\times\vec{G}\right) = \vec{G}.\nabla\times\vec{F} - \vec{F}.\nabla\times\vec{G}$
(B) $\nabla.\left(\vec{F}\times\vec{G}\right) = \vec{G}.\nabla\times\vec{F} + \vec{F}.\nabla\times\vec{G}$
(C) $\nabla.\left(g\vec{F}\right) = g\nabla.\vec{F} - \nabla g.\vec{F}$
(D) $\nabla.\left(g\vec{F}\right) = g\nabla.\vec{F} + \nabla g.\vec{F}$

35. Consider the intervals S = (0, 2] and T = [1, 3). Let S^o and T^o be the sets of interior points of S and T, respectively. Then the set of interior points of S\T is equal to-

(A) $S\backslash T^o$
(B) $S\backslash T$
(C) $S^o\backslash T^o$
(D) $S^o\backslash T$

36. Let $\{a_n\}$ be the sequence given by

$$a_n = \max\left\{\sin\left(\frac{n\pi}{3}\right), \cos\left(\frac{n\pi}{3}\right)\right\}, n \geq 1.$$

Then which of the following statements is/are TRUE about the subsequences $\{a_{6n-1}\}$ and $\{a_{6n+4}\}$?

(A) Both the subsequences are convergent
(B) Only one of the subsequences is convergent
(C) $\{a_{6n-1}\}$ converges to $-\frac{1}{2}$
(D) $\{a_{6n+4}\}$ converges to $\frac{1}{2}$

37. Let $f(x) = \cos(|\pi - x|) + (x - \pi)\sin|x|$ and $g(x) = x^2$ for $x \in \mathbb{R}$.

If $h(x) = f(g(x))$, then

(A) h is not differentiable at $x = 0$

(B) $h'\left(\sqrt{\pi}\right) = 0$

(C) $h''(x) = 0$ has a solution in $(-\pi, \pi)$

(D) there exists $x_0 \in (-\pi, \pi)$ such that $h(x_0) = x_0$

38. Let $f: \left(0, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by

$$f(x) = (\sin x)^{\pi} - \pi \sin x + \pi.$$

Then which of the following statements is/are TRUE ?

(A) f is an increasing function

(B) f is a decreasing function

(C) $f(x) > 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$

(D) $f(x) < 0$ for some $x \in \left(0, \frac{\pi}{2}\right)$

39. Let $f(x, y) = \begin{cases} \dfrac{|x|}{|x|+|y|}\sqrt{x^4 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$. Then at (0, 0).

(A) f is continuous

(B) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y}$ does not exist

(C) $\frac{\partial f}{\partial x}$ does not exist and $\frac{\partial f}{\partial y} = 0$

(D) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

40. Let $\{a_n\}$ be the sequence of real numbers such that

$$a_1 = 1 \text{ and } a_{n+1} = a_n + a_n^2 \text{ for all } n \geq 1.$$

Then

(A) $a_4 = a_1(1 + a_1)(1 + a_2)(1 + a_3)$

(B) $\lim_{n\to\infty} \frac{1}{a^n} = 0$

(C) $\lim_{n\to\infty} \frac{1}{a_n} = 1$

(D) $\lim_{n\to\infty} a_n = 0$

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

Q.41 – Q.50 carry one mark each.

41. Let x be the 100-cycle (1 2 3......100) and let y be the transposition (49 50) in the permutation group S_{100}. Then the order of xy is _________.

42. Let W_1 and W_2 be subspaces of the real vector space $\mathbb{R}^{100}$ defined by

$W_1 = \{(x_1, x_2, ..., x_{100}); x_i = 0$ if i is divisible by 4$\}$,

$W_2 = \{(x_1, x_2, ..., x_{100}); x_i = 0$ if i is divisible by 5$\}$.

Then the dimension of $W_1 \cap W_2$ is _______.

43. Consider the following system of three linear equations in four unknowns x_1, x_2, x_3 and x_4

$x_1 + x_2 + x_3 + x_4 = 4,$

$x_1 + 2x_2 + 3x_3 + 4x_4 = 4$

$x_1 + 3x_2 + 5x_3 + kx_4 = 5,$

If the system has no solutions, then k =__________.

44. Let $\vec{F}(x, y) = -y\hat{i} + x\hat{j}$ and let c be the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

oriented counter clockwise. Then the value of $\oint_C \vec{F}.d\vec{r}$ (round off to 2 decimal places) is_____.

45. The coefficient of $\left(x - \frac{\pi}{2}\right)$ in the Taylor series expansion of the function

$$f(x) = \begin{cases} \frac{4(1-\sin x)}{2x - \pi}, & x \neq \frac{\pi}{2} \\ 0, & x = \frac{\pi}{2} \end{cases}$$

about $x = \frac{\pi}{2}$, is_______..

46. Let f: [0, 1] $\to$ R be given by

$$f(x) = \frac{\left(1 + x^{\frac{1}{3}}\right)^3 + \left(1 - x^{\frac{1}{3}}\right)^3}{8(1+x)}.$$

Then

max $\{f(x) : x \in [0, 1]\}$ – min $\{f(x) : x \in [0, 1]\}$

is _________.

47. If $g(x) = \int_{x(x-2)}^{4x-5} f(t)\,dt$, where $f(x) = \sqrt{1 + 3x^4}$ for $x \in R$ then g'(1) =_________.

48. Let f(x, y) = $\begin{cases} \dfrac{x^3+y^3}{x^2-y^2}, & x^2-y^2 \neq 0 \\ 0, & x^2-y^2=0 \end{cases}$. Then the directional derivative of f at (0, 0) in the direction of $\frac{4}{5}\hat{i}+\frac{3}{5}\hat{j}$ is_________.

49. The value of the integral $\int_{-1}^{1}\int_{-1}^{1}|x+y|dx\,dy$ (round off to 2 decimal places) is ________.

50. The volume of the solid bounded by the surfaces x = 1 – y^2 and x = y^2 – 1, and the planes z = 0 and z = 2 (round off to 2 decimal places) is__________.

Q.51-Q.60 carry two marks each.

51. The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x + 2)$ about the x-axis (round off to 2 decimal places) is _________.

52. The greatest lower bound of the set $\left\{\left(e^n+2^n\right)^{\frac{1}{n}} : n \in N\right\}$,

(round off to 2 decimal places) is ___________.

53. Let G = {n ∈ N: n ≤ 55, gcd (n, 55) = 1} be the ground under multiplication modulo 55. Let x ∈ G be such that x^2 = 26 and x > 30. Then x is equal to__________.

54. The number of critical points of the function $f(x, y)=\left(x^2+3y^2\right)e^{-\left(x^2+y^2\right)}$ is _________.

55. The number of elements in the set {x ∈ S_3: x^4 = e}, where e is the identity element of the permutation group S_3, is _________.

56. If $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}$, y, z ∈ R, is an eigenvector corresponding to a real eigenvalue of the matrix $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$

then z-y is equal to_________.

57. Let M and N be any two 4 × 4 matrices with integer entries satisfying

MN = 2 $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Then the maximum value of det(M) + det (N) is _________.

58. Let M be a 3 × 3 matrix with real entries such that $M^2 = M + 2I$, where I denotes the 3 × 3 identity matrix. If α, β and γ are eigenvalues of M such that $\alpha\beta\gamma = -4$, then $\alpha + \beta + \gamma$ is equal to________.

59. Let y(x) = xv(x) be a solution of the differential equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$.

If v(0) = 0 and v(1) = 1, then v(–2) is equal to ___________.

60. If y(x) is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0,\ y(0) = 2,\ \frac{dy}{dx}(0) = 0,$$

then y(ln 2) is (round off to 2 decimal places) equal to _________.

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
D	D	B	A	B	A	C	B	A	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	C	B	B	A	C	D	A	B	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	A	C	B	C	B	D	C	B	A

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
A,C	B,C	C,D	A,D	B,D	A	B,C,D	B,C	A,D	A,B

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	42	43	44	45	46	47	48	49	50
99	60	7	75.36	1	0.25	8	2.6	2.66	5.33
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
6.70	2.72	31,46	5	4	3	17	3	4	1.19

Mathematics (MA)

Previous Year Solved Paper 2018

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ),** wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.

NOTATION

$\mathbb{Z}_n$	Set of all residue classes modulo n
$X \setminus Y$	The set of elements from X which are not in Y.
$\mathbb{N}$	The set of natural numbers = {1,2,3, …}
$\mathbb{R}$	The set of all real numbers
S_n	Set of all permutations of the set {1, 2, ...}
$GL_n(\mathbb{R})$	Set of all n × n invertible matrices with real entries
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system respectively
M^T	Transpose of a matrix M.

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 – Q.10 carry one mark each.

1. Which one of the following is True ?

(A) Z_n is cyclic if and only if n is prime.

(B) Every proper subgroup of Z_n is cyclic.

(C) Every proper subgroup of S_4 is cyclic.

(D) If every proper subgroup of a group is cyclic, then the group is cyclic.

2. Let $a_n = \frac{b_{n+1}}{b_n}$, where $b_1 = 1$, $b_2 = 1$ and $b_{n+2} = b_n + b_{n+1}$, $n \in \mathbb{N}$. Then $\lim_{n\to\infty} a_n$ is :

(A) $\frac{1-\sqrt{5}}{2}$ (B) $\frac{1-\sqrt{3}}{2}$

(C) $\frac{1+\sqrt{3}}{2}$ (D) $\frac{1+\sqrt{5}}{2}$

3. If $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors in a vector space over $\mathbb{R}$, then which one of the following sets is also linearly independent ?

(A) $\{v_1 + v_2 - v_3, 2v_1 + v_2 + 3v_3, 5v_1 + 4v_2\}$

(B) $\{v_1 - v_2, v_2 - v_3, v_3 - v_1\}$

(C) $\{v_1 + v_2 - v_3, v_2 + v_3 - v_1, v_3 + v_1 - v_2, v_1 + v_2 + v_3\}$

(D) $\{v_1 + v_2, v_2 + 2v_3, v_3 + 3v_1\}$

4. Let a be a positive real number. If f is a continuous and even function defined on the interval $[-a, a]$, then $\int_{-a}^{a} \frac{f(x)}{1+e^x}\,dx$ is equal to :

(A) $\int_0^a f(x)\,dx$ (B) $2\int_0^a \frac{f(x)}{1+e^x}\,dx$

(C) $2\int_0^a f(x)\,dx$ (D) $2a\int_0^a \frac{f(x)}{1+e^x}\,dx$

5. The tangent plane to the surface $z = \sqrt{x^2 + 3y^2}$ at (1, 1, 2) is given by

(A) $x - 3y + z = 0$ (B) $x + 3y - 2z = 0$

(C) $2x + 4y - 3z = 0$ (D) $3x - 7y + 2z = 0$

6. In $\mathbb{R}^3$, the cosine of the acute angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z - x^2 - y^2 + 3 = 0$ at the point (2, 1, 2) is :

(A) $\frac{8}{5\sqrt{21}}$ (B) $\frac{10}{5\sqrt{21}}$

(C) $\frac{8}{3\sqrt{21}}$ (D) $\frac{10}{3\sqrt{21}}$

7. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a scalar field, $\vec{v} : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector filed and let $\vec{a} \in \mathbb{R}^3$ be a constant vector. If $\vec{r}$ represents the position vector $x\hat{i} + y\hat{j} + z\hat{k}$, then which one of the following is FALSE ?

(A) $\text{curl}(f\ \vec{v}) = \text{grad}(f) \times \vec{v} + f\ \text{curl}(\vec{v})$

(B) $\text{div}(\text{grad}(f)) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) f$

(C) $\text{curl}(\vec{a} \times \vec{r}) = 2\,|\vec{a}|\,\vec{r}$

(D) $\text{div}\left(\frac{\vec{r}}{|\vec{r}|^3}\right) = 0$, for $\vec{r} \neq \vec{0}$

8. In $\mathbb{R}^2$, the family of trajectories orthogonal to the family of asteroids $x^{2/3} + y^{2/3} = a^{2/3}$ is given by

(A) $x^{4/3} + y^{4/3} = c^{4/3}$

(B) $x^{4/3} - y^{4/3} = c^{4/3}$

(C) $x^{5/3} - y^{5/3} = c^{5/3}$

(D) $x^{2/3} - y^{2/3} = c^{2/3}$

9. Consider the vector space V over $\mathbb{R}$ of polynomial functions of degree less then or equal to 3 defined on $\mathbb{R}$. Let $T : V \to V$ be defined by $(Tf)(x) = f(x) - xf'(x)$. Then the rank of T is

(A) 1

(B) 2

(C) 3

(D) 4

10. Let $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!}$ for $n \in N$. Then which one of the following is True for the sequence $\{s_n\}_{n=1}^{\infty}$?

(A) $\{s_n\}_{n=1}^{\infty}$ converges in $\mathbb{Q}$

(B) $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence but does not converge in $\mathbb{Q}$

(C) the subsequence $\{s_{k^n}\}_{n=1}^{\infty}$ is convergent in $\mathbb{R}$, only when k is even natural number

(D) $\{s_n\}_{n=1}^{\infty}$ is not a Cauchy sequence.

Q. 11 – Q. 30 carry two marks each.

11. Let $a_n = \begin{cases} 2 + \frac{(-1)^{\frac{n-1}{2}}}{n}, & \text{if n is odd} \\ 1 + \frac{1}{2^n}, & \text{if n is even} \end{cases}$, $n \in N$.

Then which one of the following is TRUE ?

(A) $\sup\{a_n \mid n \in \mathbb{N}\} = 3$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$

(B) $\lim\inf (a_n) = \lim\sup (a_n) = \frac{3}{2}$

(C) $\sup\{a_n \mid n \in \mathbb{N}\} = 2$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$

(D) $\lim\inf (a_n) = 1$ and $\lim\sup (a_n) = 3$

12. Let a, b, c $\in \mathbb{R}$. Which of the following values of a, b, c do NOT result in the convergence of the series

$$\sum_{n=3}^{\infty} \frac{a^n}{n^b (\log_e n)^c} \ ?$$

(A) $|a| < 1,\ b \in \mathbb{R},\ c \in \mathbb{R}$ (B) $a = 1,\ b > 1,\ c \in \mathbb{R}$

(C) $a = 1,\ b \geq 0,\ c < 1$ (D) $a = -1,\ b \geq 0,\ c > 0$

13. Let $a_n = n + \frac{1}{n}$, n $\in \mathbb{N}$. Then the sum of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_{n+1}}{n!}$ is :

(A) $e^{-1} - 1$ (B) e^{-1}

(C) $1 - e^{-1}$ (D) $1 + e^{-1}$

14. Let $a_n = \frac{(-1)^n}{\sqrt{1+n}}$ and let $c_n = \sum_{k=0}^{n} a_{n-k} a_k$, where n $\in \mathbb{N} \cup \{0\}$. Then which one of the following is true ?

(A) Both $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$ are convergent

(B) $\sum_{n=0}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} c_n$ is not convergent

(C) $\sum_{n=1}^{\infty} c_n$ is convergent but $\sum_{n=0}^{\infty} a_n$ is not convergent

(D) Neither $\sum_{n=0}^{\infty} a_n$ nor $\sum_{n=1}^{\infty} c_n$ is convergent.

15. Suppose that f, g : $\mathbb{R} \to \mathbb{R}$ are differentiable functions such that f is strictly increasing and g is strictly decreasing. Define p(x) = f(g(x)) and q(x) = g(f(x)), $\forall$ x $\in \mathbb{R}$. Then, for t > 0, the sign of

$\int_0^t p'(x)(q'(x) - 3)\, dx$ is :

(A) positive (B) negative

(C) dependent on t (D) dependent on f and g

16. For x $\in \mathbb{R}$, let $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then which one of the following is false ?

(A) $\lim_{x \to 0} \frac{f(x)}{x} = 0$

(B) $\lim_{x \to 0} \frac{f(x)}{x^2} = 0$

(C) $\frac{f(x)}{x^2}$ has infinitely many maxima and minima on the interval (0, 1)

(D) $\frac{f(x)}{x^4}$ is continuous at x = 0 but not differentiable at x = 0

17. Let $f(x, y) = \begin{cases} \dfrac{xy}{(x^2+y^2)^\alpha}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

Then which one of the following is true for f at the point (0, 0) ?

(A) For α = 1, f is continuous but not differentiable.

(B) For $\alpha = \frac{1}{2}$, f is continuous and differentiable.

(C) For $\alpha = \frac{1}{4}$, f is continuous and differentiable.

(D) For $\alpha = \frac{3}{4}$, f is neither continuous nor differentiable.

18. Let a, b $\in \mathbb{R}$ and let f : $\mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. If $z = e^u f(v)$, where $u = ax + by$ and $v = ax - by$, then which one of the following is true ?

(A) $b^2z_{xx} - a^2z_{yy} = 4a^2b^2e^uf'(v)$ (B) $b^2z_{xx} - a^2z_{yy} = -4e^uf'(v)$

(C) $bz_x + az_y = abz$ (D) $bz_x + az_y = -abz$

19. Consider the region D in the yz plane bounded by the line $y = \frac{1}{2}$ and the curve $y^2 + z^2 = 1$, where $y \geq 0$. If the region D is revolved about the z-axis in $\mathbb{R}^3$, then the volume of the resulting solid is :

(A) $\frac{\pi}{\sqrt{3}}$ (B) $\frac{2\pi}{\sqrt{3}}$

(C) $\frac{\pi\sqrt{3}}{2}$ (D) $\pi\sqrt{3}$

20. If $\vec{F}(x, y) = (3x - 8y)\hat{i} + (4y - 6xy)\hat{j}$ for $(x, y) \in \mathbb{R}^2$, then $\oint_C \vec{F} \cdot d\vec{r}$, where C is the boundary of the triangular region bounded by the lines x = 0, y = 0 and x + y = 1 oriented in the anti-clockwise direction, is :

(A) $\frac{5}{2}$ (B) 3

(C) 4 (D) 5

21. Let U, V and W be finite dimensional real vector spaces, T : U $\to$ V, S : V $\to$ W and P : W $\to$ U be linear transformations. If range (ST) = nullspace (P), nullspace (ST) = range (P) and rank (T) = rank (S), then which one of the following is true ?

(A) nullity of T = nullity of S

(B) dimension of U $\neq$ dimension of W

(C) If dimension of V = 3, dimension of U = 4, then P is not identically zero

(D) If dimension of V = 4, dimension of U = 3 and T is one-one, then P is identically zero

22. Let y(x) be the solution of the differential equation $\frac{dy}{dx} + y = f(x)$, for x ≥ 0, y(0) = 0, where $f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & x \ge 1 \end{cases}$. Then y(x) =

(A) $2(1 - e^{-x})$ when $0 \le x < 1$ and $2(e - 1)e^{-x}$ when $x \ge 1$

(B) $2(1 - e^{-x})$ when $0 \le x < 1$ and 0 when $x \ge 1$

(C) $2(1 - e^{-x})$ when $0 \le x < 1$ and $2(1 - e^{-1})e^{-x}$ when $x \ge 1$

(D) $2(1 - e^{-x})$ when $0 \le x < 1$ and $2e^{1-x}$ when $x \ge 1$

23. An integrating factor of the differential equation $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}(x + xy^2)dy = 0$ is :

(A) x^2 (B) $3 \log_e x$

(C) x^3 (D) $2 \log_e x$

24. A particular integral of the differential equation $y'' + 3y' + 2y = e^{e^x}$ is :

(A) $e^{e^x}e^{-x}$ (B) $e^{e^x}e^{-2x}$

(C) $e^{e^x}e^{2x}$ (D) $e^{e^x}e^{x}$

25. Let G be a group satisfying the property that $f : G \to \mathbb{Z}_{221}$ is a homomorphism implies f(g) = 0, ∀ g ∈ G. Then a possible group G is :

(A) $\mathbb{Z}_{21}$ (B) $\mathbb{Z}_{51}$

(C) $\mathbb{Z}_{91}$ (D) $\mathbb{Z}_{119}$

26. Let H be the quotient group $\mathbb{Q}/\mathbb{Z}$. Consider the following statements.

I. Every cyclic subgroup of H is finite.

II. Every finite cyclic group is isomorphic to a subgroup of H.

Which one of the following holds ?

(A) I is true but II is false (B) II is true but I is false

(C) both I and II are true (D) neither I nor II is true

27. Let I denote the 4 × 4 identity matrix. If the roots of the characteristic polynomial of a 4 × 4 matrix M are $\pm\sqrt{\frac{1 \pm \sqrt{5}}{2}}$, then M^8 =

(A) $I + M^2$ (B) $2I + M^2$

(C) $2I + 3M^2$ (D) $3I + 2M^2$

28. Consider the group $\mathbb{Z}^2 = \{(a, b) \mid a, b \in \mathbb{Z}\}$ under component-wise addition. Then which of the following is a subgroup of $\mathbb{Z}^2$?

(A) $\{(a, b) \in \mathbb{Z}^2 \mid ab = 0\}$

(B) $\{(a, b) \in \mathbb{Z}^2 \mid 3a + 2b = 15\}$

(C) $\{(a, b) \in \mathbb{Z}^2 \mid 7 \text{ divides } ab\}$

(D) $\{(a, b) \in \mathbb{Z}^2 \mid 2 \text{ divides } a \text{ and } 3 \text{ divides } b\}$

29. Let $f : \mathbb{R} \to \mathbb{R}$ be a function and let J be a bounded open interval in $\mathbb{R}$. Define

$$W(f, J) = \sup \{f(x) \mid x \in J\} - \inf \{f(x) \mid x \in J\}.$$

Which one of the following is false ?

(A) $W(f, J_1) \leq W(f, J_2)$ if $J_1 \subset J_2$

(B) If f is a bounded function in J and $J \supset J_1 \supset J_2 \ldots\ldots \supset J_n \supset \ldots\ldots$ such that the length of the interval J_n tends to 0 and $n \to \infty$, then $\lim\limits_{n\to\infty} W(f, J_n) = 0$

(C) If f is discontinuous at a point $a \in J$, then $W(f, J) \neq 0$

(D) If f is continuous at a point $a \in J$, then for any given $\in > 0$ there exists an interval $I \subset J$ such that $W(f, I) < \in$

30. For $x > \dfrac{-1}{2}$, let $f_1(x) = \dfrac{2x}{1+2x}$, $f_2(x) = \log_e(1 + 2x)$ and $f_3(x) = 2x$. Then which one of the following is true ?

(A) $f_3(x) < f_2(x) < f_1(x)$ for $0 < x < \dfrac{\sqrt{3}}{2}$

(B) $f_1(x) < f_3(x) < f_2(x)$ for $x > 0$

(C) $f_1(x) + f_2(x) < \dfrac{f_3(x)}{2}$ for $x > \dfrac{\sqrt{3}}{2}$

(D) $f_2(x) < f_1(x) < f_3(x)$ for $x > 0$

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

31. Let $f : \mathbb{R}\backslash\{0\} \to \mathbb{R}$ be defined by $f(x) = x + \dfrac{1}{x^3}$. On which of the following interval(s) is f one-one?

(A) $(-\infty, -1)$ (B) $(0, 1)$

(C) $(0, 2)$ (D) $(0, \infty)$

32. The solution(s) of the differential equation $\dfrac{dy}{dx} = (\sin 2x)y^{1/3}$ satisfying $y(0) = 0$ is (are)

(A) $y(x) = 0$ (B) $y(x) = -\sqrt{\dfrac{8}{27}\sin^3 x}$

(C) $y(x) = \sqrt{\dfrac{8}{27}\sin^3 x}$ (D) $y(x) = \sqrt{\dfrac{8}{27}\cos^3 x}$

33. Suppose f, g, h are permutations of the set $\{\alpha, \beta, \gamma, \delta\}$, where

f interchanges α and β but fixes γ and δ,

g interchanges β and γ but fixes α and δ,

h interchanges γ and δ but fixes α and β.

Which of the following permutations interchange(s) α and δ but fix(es) β and γ ?

(A) f o g o h o g o f (B) g o h o f o h o g

(C) g o f o h o f o g (D) h o g o f o g o h

34. Let P and Q be two non-empty disjoint subsets of $\mathbb{R}$. Which of the following is (are) false ?

(A) If P and Q are compact, then $P \cup Q$ is also compact

(B) If P and Q are not connected, then $P \cup Q$ is also not connected

(C) If $P \cup Q$ and P are closed, the Q is closed

(D) If $P \cup Q$ and P are open, then Q is open

35. Let $\mathbb{C}^* = \mathbb{C}\backslash\{0\}$ denote the group of non-zero complex numbers under multiplication. Suppose $Y_n = \{z \in \mathbb{C} \mid z^n = 1\}$, $n \in \mathbb{N}$. Which of the following is (are) subgroup(s) of $\mathbb{C}^*$?

(A) $\cup_{n=1}^{100} Y_n$ (B) $\cup_{n=1}^{\infty} Y_{2^n}$

(C) $\cup_{n=100}^{\infty} Y_n$ (D) $\cup_{n=1}^{\infty} Y_n$

36. Suppose $\alpha, \beta, \gamma \in \mathbb{R}$. Consider the following system of linear equations.
$x + y + z = \alpha$, $x + \beta y + z = \gamma$, $x + y + \alpha z = \beta$. If this system has at least one solution, then which of the following statements is (are) true ?

(A) If $\alpha = 1$ then $\gamma = 1$ (B) If $\beta = 1$ then $\gamma = \alpha$

(C) If $\beta \neq 1$ then $\alpha = 1$ (D) If $\gamma = 1$ then $\alpha = 1$

37. Let $m, n \in \mathbb{N}$, $m < n$, $P \in M_{n\times m}(\mathbb{R})$, $Q \in M_{m\times n}(\mathbb{R})$. Then which of the following is (are) Not possible?

(A) rank (PQ) = n

(B) rank (QP) = m

(C) rank (PQ) = m

(D) rank (QP) = $\left\lceil \frac{m+n}{2} \right\rceil$, the smallest integer larger than or equal to $\frac{m+n}{2}$

38. If $\vec{F}(x,y,z) = (2x+3yz)\hat{i} + (3xz+2y)\hat{j} + (3xy+2z)\hat{k}$ for $(x, y, z) \in \mathbb{R}^3$, then which among the following is (are) true ?

(A) $\nabla \times \vec{F} = \vec{0}$

(B) $\oint_C \vec{F} \cdot d\vec{r} = 0$ along any simple closed curve C

(C) There exists a scalar function $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla \cdot \vec{F} = \phi_{xx} + \phi_{yy} + \phi_{zz}$

(D) $\nabla \cdot \vec{F} = 0$

39. Which of the following subsets of $\mathbb{R}$ is (are) connected ?

(A) $\{x \in \mathbb{R} \mid x^2 + x > 4\}$ (B) $\{x \in \mathbb{R} \mid x^2 + x < 4\}$

(C) $\{x \in \mathbb{R} \mid |x| < |x - 4|\}$ (D) $\{x \in \mathbb{R} \mid |x| > |x - 4|\}$

40. Let S be a subset of $\mathbb{R}$ such that 2018 is an interior point of S. Which of the following is (are) true ?

(A) S contains an interval

(B) There is a sequence in S which does not converge to 2018

(C) There is an element $y \in S$, $y \neq 2018$ such that y is also an interior point of S

(D) There is point $z \in S$, such that $|z - 2018| = 0.002018$

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41. The order of the element (1 2 3)(2 4 5)(4 5 6) in the group S_6 is _______ .

42. Let $\phi(x, y, z) = 3y^2 + 3yz$ for $(x, y, z) \in \mathbb{R}^3$. Then the absolute value of the directional derivative of ϕ in the direction of the line $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{-2}$, at the point (1, – 2, 1) is _______ .

43. Let $f(x) = \sum_{n=0}^{\infty}(-1)^n x(x-1)^n$ for $0 < x < 2$. Then the value of $f\left(\frac{\pi}{4}\right)$ is _______ .

44. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = \begin{cases} \dfrac{x^2y(x-y)}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Then $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$ at the point (0, 0) is _______ .

45. Let $f(x,y) = \sqrt{x^3y}\sin\left(\frac{\pi}{2}e^{\left(\frac{y}{x}-1\right)}\right) + xy\cos\left(\frac{\pi}{3}e^{\left(\frac{x}{y}-1\right)}\right)$ for $(x, y) \in \mathbb{R}^2$, $x > 0$, $y > 0$. Then $f_x(1, 1) + f_y(1, 1)$ = _______ .

46. Let $f : [0, \infty) \to [0, \infty)$ be continuous on $[0, \infty)$ and differentiable on $(0, \infty)$. If $f(x) = \int_0^x \sqrt{f(t)}\,dt$, then f(6) = _______ .

47. Let $a_n = \frac{(1+(-1)^n)}{2^n} + \frac{(1+(-1)^{n-1})}{3^n}$. Then the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ about x = 0 is _______ .

48. Let A_6 be the group of even permutations of 6 distinct symbols. Then the number of elements of order 6 in A_6 is _______ .

49. Let W_1 be the real vector space of all 5 × 2 matrices such that the some of the entries in each row is zero. Let W_2 be the real vector space of all 5 × 2 matrices such that the sum of the entries in each column is zero. Then the dimension of the space $W_1 \cap W_2$ is _______ .

50. The coefficient of x^4 in the power series expansion of $e^{\sin x}$ about x = 0 is _______ (correct up to three decimal places).

Q. 51 – Q. 60 carry two marks each.

51. Let $a_k = (-1)^{k-1}$, $s_n = a_1 + a_2 + ... + a_n$ and $\sigma_n = (s_1 + s_2 + ... + s_n)/n$, where $k, n \in \mathbb{N}$. Then $\lim_{n\to\infty} \sigma_n$ is _______ . (correct up to one decimal places).

52. Let $f:\mathbb{R} \to \mathbb{R}$ be such that f" is continuous on $\mathbb{R}$ and $f(0) = 1$, $f'(0) = 0$ and $f''(0) = -1$. Then $\lim_{x\to\infty}\left(f\left(\sqrt{\frac{2}{x}}\right)\right)^x$ is _______ (correct up to three decimal places).

53. Suppose x, y, z are positive real number such that $x + 2y + 3z = 1$. If M is the maximum value of xyz^2, then the value of $\frac{1}{M}$ is _______ .

54. If the volume of the solid in $\mathbb{R}^3$ bounded by the surfaces

$$x = -1,\ x = 1,\ y = -1,\ y = 1,\ z = 2,\ y^2 + z^2 = 2$$

is $\alpha - \pi$, then α = _______ .

55. If $a = \int_{\pi/6}^{\pi/3} \frac{\sin t + \cos t}{\sqrt{\sin 2t}}\,dt$, then the value of $\left(2\sin\frac{a}{2} + 1\right)^2$ is _______ .

56. The value of the integral

$$\int_0^1 \int_x^1 y^4 e^{xy^2}\,dy\,dx$$

is _______ . (correct up to three decimal places).

57. Suppose $Q \in M_{3\times3}(\mathbb{R})$ is a matrix of rank 2. Let $T : M_{3\times3}(\mathbb{R}) \to M_{3\times3}(\mathbb{R})$ be the linear transformation defined by $T(P) = QP$. Then the rank of T is _______ .

58. The area of the parametrized surface

$$S = \left\{((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u) \in \mathbb{R}^3 \mid 0 \le u \le \frac{\pi}{2}, 0 \le v \le \frac{\pi}{2}\right\}$$

is _______ (correct up to two decimal places).

59. If x(t) is the solution to the differential equation $\frac{dx}{dt} = x^2t^3 + xt$, for $t > 0$, satisfying $x(0) = 1$, then the value of $x\left(\sqrt{2}\right)$ is _______ (correct up to two decimal places).

60. If $y(x) = v(x) \sec x$ is the solution of $y'' - (2\tan x)y' + 5y = 0$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, satisfying $y(0) = 0$ and $y'(0) = \sqrt{6}$, then $v\left(\frac{\pi}{6\sqrt{6}}\right)$ is _______ . (correct up to two decimal places).

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	D	D	A	B	C	C	B	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	C	D	B	A	D	C	A	C	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	A	C	B	A	C	C	D	B	MTA

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
B	A,B,C	A,D	B,C,D	B,C,D	A,B	A,D	A,B,C	B,C,D	A,B,C

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
4	7	1	1	3	9	2	0	4	–0.125
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
0.5	0.368	1152	6	3	0.239	6	6.505	–2.718	0.5

Mathematics (MA)

Previous Year Solved Paper 2017

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains **Numerical Answer Type Questions (NAT).** For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In Section – A (MCQ), wrong answer will result in NEGATIVE marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In Section – B (MSQ), there is NO NEGATIVE and NO PARTIAL marking provisions.There is NO NEGATIVE marking in Section – C (NAT) as well.

NOTATION

$\mathbb{Z}_n$	Set of all residue classes modulo n
X\Y	The set of elements from X which are not in Y.
$\mathbb{N}$	The set of natural numbers = {1,2,3, …}
$\mathbb{R}$	The set of all real numbers
S_n	Set of all permutations of the set {1, 2, ...}
$GL_n(\mathbb{R})$	Set of all n × n invertible matrices with real entries
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system respectively
M^T	Transpose of a matrix M.

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 to Q. 10 carry one mark each.

1. Consider the function $f(x, y) = 5 - 4 \sin x + y^2$ for $0 < x < 2\pi$ and $y \in \mathbb{R}$. The set of critical points of f(x, y) consists of

(A) a point of local maximum and a point of local minimum

(B) a point of local maximum and a saddle point

(C) a point of local maximum, a point of local minimum and a saddle point

(D) a point of local minimum and a saddle point

2. Let $\varphi : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that φ' is strictly increasing with $\varphi'(1) = 0$. Let α and β denote the minimum and maximum values of $\varphi(x)$ on the interval [2, 3], respectively. Then which one of the following is TRUE ?

(A) $\beta = \varphi(3)$ (B) $\alpha = \varphi(2.5)$

(C) $\beta = \varphi(2.5)$ (D) $\alpha = \varphi(3)$

3. The number of generators of the additive group $\mathbb{Z}_{36}$ is equal to

(A) 6 (B) 12

(C) 18 (D) 36

4. $\lim\limits_{n\to\infty} \frac{\pi}{n}\sum\limits_{k=1}^{n} \sin\left(\frac{\pi}{2} + \frac{5\pi}{2}.\frac{k}{n}\right) =$

(A) $\frac{2\pi}{5}$ (B) $\frac{5}{2}$

(C) $\frac{2}{5}$ (D) $\frac{5\pi}{2}$

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $g(u, v) = f(u^2 - v^2)$, then

$$\frac{\partial^2 g}{\partial u^2} + \frac{\partial^2 g}{\partial v^2} =$$

(A) $4(u^2 - v^2)\, f''(u^2 - v^2)$

(B) $4(u^2 + v^2)\, f''(u^2 - v^2)$

(C) $2f'(u^2 - v^2) + 4(u^2 - v^2)\, f''(u^2 - v^2)$

(D) $2(u - v)^2\, f''(u^2 - v^2)$

6. $\int_0^1\int_x^1 \sin(y^2)dy\, dx =$

(A) $\frac{1+\cos 1}{2}$ (B) $1 - \cos 1$

(C) $1 + \cos 1$ (D) $\frac{1-\cos 1}{2}$

7. Let $f_1(x)$, $f_2(x)$, $g_1(x)$, $g_2(x)$ be differentiable functions on $\mathbb{R}$. Let $F(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$ be the determinant of the matrix $\begin{bmatrix} f_1(g) & f_2(x) \\ g_1(x) & g_2(x) \end{bmatrix}$. Then F'(x) is equal to

(A) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2'(x) & g_2(x) \end{vmatrix}$

(B) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(C) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} - \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(D) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

8. Let $f(x) = \dfrac{x + |x|(1+x)}{x}\sin\left(\dfrac{1}{x}\right), \quad x \neq 0.$

Write $L = \lim\limits_{x\to 0^-} f(x)$ and $R = \lim\limits_{x\to 0^+} f(x)$. Then which one of the following is TRUE ?

(A) L exists but R does not exist

(B) L does not exist but R exists

(C) Both L and R exist

(D) Neither L nor R exists

9. If $\lim_{T\to\infty} \int_0^T e^{-x^2}\,dx = \dfrac{\sqrt{\pi}}{2}$, then $\lim\limits_{T\to\infty} \int_0^T x^2 e^{-x^2}\,dx =$

(A) $\dfrac{\sqrt{\pi}}{4}$

(B) $\dfrac{\sqrt{\pi}}{2}$

(C) $\sqrt{2\pi}$

(D) $2\sqrt{\pi}$

10. If $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ (1-x)(px+q) & \text{if } x \geq 0 \end{cases}$

satisfies the assumptions of Rolle's theorem in the interval [–1,1], then the ordered pair (p,q) is

(A) (2, –1)

(B) (–2, –1)

(C) (–2, 1)

(D) (2, 1)

Q.11 – Q.30 carry two marks each.

11. The flux of the vector field

$$\vec{F} = \left(2\pi x + \frac{2x^2y^2}{\pi}\right)\hat{i} + \left(2\pi xy - \frac{4y}{\pi}\right)\hat{j}$$

along the outward normal, across the ellipse $x^2 + 16y^2 = 4$ is equal to

(A) $4\pi^2 - 2$

(B) $2\pi^2 - 4$

(C) $\pi^2 - 2$

(D) 2π

12. Let $\mathcal{M}$ be the set of all invertible 5 × 5 matrices with entries 0 and 1. For each $M \in \mathcal{M}$, let $n_1(M)$ and $n_0(M)$ denote the number of 1's and 0's in M, respectively. Then

$$\min_{M\in\mathcal{M}} |n_1(M) - n_0(M)| =$$

(A) 1

(B) 3

(C) 5

(D) 15

13. Let M = $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 0 & 1 \end{bmatrix}$ and x = $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Then $\lim_{n\to\infty} M^n x$

(A) does not exist
(B) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
(C) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$
(D) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

14. Let $\vec{F} = (3 + 2xy)\,\hat{i} + (x^2 - 3y^2)\,\hat{j}$ and let L be the curve

$$\vec{r}(t) = e^t \sin t\,\hat{i} + e^t \cos t\,\hat{j}, \quad 0 \le t \le \pi.$$

Then

$$\int_L \vec{F}.d\vec{r} =$$

(A) $e^{-3\pi} + 1$
(B) $e^{-6\pi} + 2$
(C) $e^{6\pi} + 2$
(D) $e^{3\pi} + 1$

15. The line integral of the vector field

$$\vec{F} = zx\ \hat{i} + xy\ \hat{j} + yz\ \hat{k}$$

along the boundary of the triangle with vertices (1,0,0), (0,1,0) and (0,0,1), oriented anti-clockwise, when viewed from the point (2,2,2) is

(A) $\frac{-1}{2}$
(B) –2
(C) $\frac{1}{2}$
(D) 2

16. The area of the surface z = $\frac{xy}{3}$ intercepted by the cylinder $x^2 + y^2 \le 16$ lies in the interval

(A) $(20\pi, 22\pi]$
(B) $(22\pi, 24\pi]$
(C) $(24\pi, 26\pi]$
(D) $(26\pi, 28\pi]$

17. For a > 0, b > 0, let $\vec{F} = \frac{x\hat{j} - y\hat{i}}{b^2x^2 + a^2y^2}$ be a planar vector field. Let

$$C = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = a^2 + b^2\}$$

be the circle oriented anti-clockwise. Then $\oint_C F.dr =$

(A) $\frac{2\pi}{ab}$
(B) 2π
(C) $2\pi ab$
(D) 0

18. The flux of $\vec{F} = y\hat{i} - x\hat{j} + z^2\hat{k}$ along the outward normal, across the surface of the solid

$$\left\{(x, y, z) \in \mathbb{R}^3 \mid 0 \le x \le 1, 0 \le y \le 1, \ 0 \le z \le \sqrt{2 - x^2 - y^2}\right\}$$

is equal to

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$

(C) $\frac{8}{3}$ (D) $\frac{4}{3}$

19. Let f : $\mathbb{R} \to \mathbb{R}$ be a differential function such that f(2) = 2 and

$$|f(x) - f(y)| \le 5(|x - y|)^{3/2}$$

for all $x \in \mathbb{R}$, $y \in \mathbb{R}$. Let $g(x) = x^3 f(x)$. Then g'(2) =

(A) 5 (B) $\frac{15}{2}$

(C) 12 (D) 24

20. Let f : $\mathbb{R} \to [0, \infty)$ be a continuous function. Then which one of the following is NOT TRUE ?

(A) There exists $x \in \mathbb{R}$ such that $f(x) = \frac{f(0) + f(1)}{2}$

(B) There exists $x \in \mathbb{R}$ such that $f(x) = \sqrt{f(-1)f(1)}$

(C) There exists $x \in \mathbb{R}$ such that $f(x) = \int_{-1}^{1} f(t)dt$

(D) There exists $x \in \mathbb{R}$ such that $f(x) = \int_{0}^{1} f(t)dt$

21. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{(-3)^{n+2}} \frac{(4x - 12)^n}{n^2 + 1}$ is

(A) $\frac{10}{4} \le x < \frac{14}{4}$ (B) $\frac{9}{4} \le x < \frac{15}{4}$

(C) $\frac{10}{4} \le x \le \frac{14}{4}$ (D) $\frac{9}{4} \le x \le \frac{15}{4}$

22. Let P_3 denote the real vector space of all polynomials with real coefficients of degree at most 3. Consider the map $T : P_3 \to P_3$ given by T(p(x)) = p"(x) + p(x). Then

(A) T is neither one-one nor onto (B) T is both one-one and onto

(C) T is one-one but not onto (D) T is onto but not one-one

23. Let $f(x, y) = \frac{x^2 y}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$. Then

(A) $\frac{\partial f}{\partial x}$ and f are bounded.

(B) $\frac{\partial f}{\partial x}$ is bounded and f is unbounded.

(C) $\frac{\partial f}{\partial x}$ is unbounded and f is bounded.

(D) $\frac{\partial f}{\partial x}$ and f are unbounded.

24. Let S be an infinite subset of $\mathbb{R}$ such that $S\setminus\{\alpha\}$ is compact for some $\alpha \in S$. Then which one of the following is TRUE ?

(A) S is a connected set.

(B) S contains no limit points.

(C) S is a union of open intervals.

(D) Every sequence in S has a subsequence converging to an element in S.

25. $\sum_{n=1}^{\infty} \tan^{-1}\frac{2}{n^2} =$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$ (D) π

26. Let $0 < a_1 < b_1$. For $n \geq 1$, define

$$a_{n+1} = \sqrt{a_n b_n} \text{ and } b_{n+1} = \frac{a_n + b_n}{2}$$

Then which one of the followings is NOT TRUE ?

(A) Both $\{a_n\}$ and $\{b_n\}$ converge, but the limits are not equal.

(B) Both $\{a_n\}$ and $\{b_n\}$ converge and the limits are equal.

(C) $\{b_n\}$ is decreasing sequence.

(D) $\{a_n\}$ is an increasing sequence.

27. $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{9}} + \ldots + \frac{1}{\sqrt{3n}+\sqrt{3n+3}}\right) =$

(A) $1+\sqrt{3}$ (B) $\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{1+\sqrt{3}}$

28. Which one of the following is TRUE ?

(A) Every sequence that has a convergent subsequence is a Cauchy sequence.

(B) Every sequence that has a convergent subsequence is a bounded sequence.

(C) The sequence $\{\sin n\}$ has a convergent subsequence.

(D) The sequence $\left\{n\cos\frac{1}{n}\right\}$ has a convergent subsequence.

29. A particular integral of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x}\sin x$ is

(A) $\frac{e^{2x}}{10}(3\cos x - 2\sin x)$ (B) $-\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(C) $-\frac{e^{2x}}{5}(2\cos x + \sin x)$ (D) $\frac{e^{2x}}{5}(2\cos x - \sin x)$

30. Let y(x) be the solution of the differential equation

$$(xy + y + e^{-x})dx + (x + e^{-x})dy = 0$$

satisfying y(0) = 1. Then y(–1) is equal to

(A) $\frac{e}{e-1}$ (B) $\frac{2e}{e-1}$

(C) $\frac{e}{1-e}$ (D) 0

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

31. For $\alpha, \beta \in \mathbb{R}$, define the map $\varphi_{\alpha,\beta} : \mathbb{R} \to \mathbb{R}$ by $\varphi_{\alpha,\beta}(x) = \alpha x + \beta$. Let

$$G = \left[\varphi_{\alpha,\beta} \mid (\alpha,\beta) \in \mathbb{R}^2\right]$$

For f, g $\in$ G, define g$\circ$f $\in$ G by (g$\circ$f)(x) = g(f(x)). Then which of the following statements is/are TRUE ?

(A) The binary operation $\circ$ is associative.

(B) The binary operation $\circ$ is commutative.

(C) For every $(\alpha, \beta) \in \mathbb{R}^2$, $\alpha \neq 0$ there exists $(a, b) \in \mathbb{R}^2$ such that $\varphi_{\alpha,\beta} \circ \varphi_{a,b} = \varphi_{1.0.}$

(D) (G, $\circ$) is a group.

32. The volume of the solid $\left\{(x,y,z) \in \mathbb{R}^3 \middle| 1 \le x \le 2,\ \ 0 \le y \le \frac{2}{x},\ \ 0 \le z \le x\right\}$ is expressible as

(A) $\int_1^2 \int_0^{2/x} \int_0^x dz\, dy\, dx$ (B) $\int_1^2 \int_0^x \int_0^{2/x} dy\, dz\, dx$

(C) $\int_0^2 \int_1^z \int_0^{2/x} dy\, dx\, dz$ (D) $\int_0^2 \int_{\max\{z,1\}}^2 \int_0^{2/x} dy\, dx\, dz$

33. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function. Then which of the following statements is/are TRUE ?

(A) If f is differentiable at (0, 0), then all directional derivatives of f exist at (0, 0).

(B) If all directional derivatives of f exist at (0, 0), then f is differentiable at (0, 0).

(C) If all directional derivatives of f exist at (0, 0), then f is continuous at (0, 0).

(D) If the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are continuous in a disc centered at (0, 0) then f is differentiable at (0, 0).

34. If X and Y are n × n matrices with real entries, then which of the following is/are TRUE ?

(A) If $P^{-1}XP$ is diagonal for some real invertible matrix P, then there exists a basis for $\mathbb{R}^n$ consisting of eigenvectors of X.

(B) If X is diagonal with distinct diagonal entries and XY = YX, then Y is also diagonal.

(C) If X^2 is diagonal, then X is diagonal.

(D) If X is a diagonal and XY = YX for all Y, then X = λI for some $\lambda \in \mathbb{R}$

35. Let G be a group of order 20 in which the conjugacy classes have sizes 1, 4, 5, 5, 5. Then which of the followings is/are TRUE ?

(A) G contains a normal subgroup of order 5.

(B) G contains a non-normal subgroup of order 5.

(C) G contains a subgroup of order 10.

(D) G contains a normal subgroup of order 4.

36. Let $\{x_n\}$ be a real sequence such that $7x_{n+1} = x_n^3 + 6$ for $n \geq 1$. Then which of the following statements is/are TRUE ?

(A) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 1.

(B) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 2.

(C) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to 1.

(D) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to –3.

37. Let S be the set of all rational numbers in (0, 1). Then which of the following statements is/are TRUE ?

(A) S is a closed subset of $\mathbb{R}$

(B) S is not a closed subset of $\mathbb{R}$

(C) S is an open subset of $\mathbb{R}$

(D) Every $x \in (0, 1)\backslash S$ is a limit point of S.

38. Let M be an n × n matrix with real entries such that $M^3 = I$. Suppose that $Mv \neq v$ for any non-zero vector v. Then which of the following statements is/are TRUE ?

(A) M has real eigenvalues

(B) $M + M^{-1}$ has real eigenvalues

(C) n is divisible by 2

(D) n is divisible by 3

39. Let y(x) be the solution of the differential equation

$$\frac{dy}{dx} = (y-1)(y-3)$$

satisfying the condition y(0) = 2. Then which of the following is/are TRUE ?

(A) The function y(x) is not bounded above

(B) The function y(x) is bounded

(C) $\lim\limits_{x\to+\infty} y(x) = 1$

(D) $\lim\limits_{x\to-\infty} y(x) = 3$

40. Let $k, \ell \in \mathbb{R}$ be such that every solution of

$$\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} = \ell y = 0$$

satisfies $\lim\limits_{x\to\infty} y(x) = 0$. Then

(A) $3k^2 + \ell < 0$ and $k > 0$

(B) $k^2 + \ell > 0$ and $k < 0$

(C) $k^2 - \ell \leq 0$ and $k > 0$

(D) $k^2 - \ell > 0$, $k > 0$ and $\ell > 0$

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

Q.41 – Q.50 carry one mark each.

41. If the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 = c_1$, $c_1 > 0$, are given by $y = c_2x^{\alpha}$, $c_2 \in \mathbb{R}$, then α = _________.

42. Let G be a subgroup of $GL_2(\mathbb{R})$ generated by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$. Then the order of G is ______.

43. Consider the permutations $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 3 & 7 & 8 & 6 & 1 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 3 & 1 & 7 & 6 & 8 & 2 \end{pmatrix}$ in S_8. The number of $\eta \in S_8$ such that $\eta^{-1}\sigma\eta = \tau$ is equal to ________.

44. Let P be the point on the surface $z = \sqrt{x^2 + y^2}$ closet to the point (4,2,0). Then the square of the distance between the origin and P is ______.

45. $\left(\int_0^1 x^4(1-x)^5\,dx\right)^{-1}$ = ________ .

46. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Let M be the matrix whose columns are v_1, v_2, $2v_1 - v_2$, $v_1 + 2v_2$ in that order. Then the number of linearly independent solutions of the homogeneous system of linear equations $Mx = 0$ is __________.

47. $\frac{1}{2\pi}\left(\frac{\pi^3}{1!3} - \frac{\pi^5}{3!5} + \frac{\pi^7}{5!7} - \ldots + \frac{(-1)^{n-1}\pi^{2n+1}}{(2n-1)!\,(2n+1)} + \ldots\right)$ = ________ .

48. Let P be a 7 × 7 matrix of rank 4 with real entries. Let $a \in \mathbb{R}^7$ be a column vector. Then the rank of $P + aa^T$ is at least ________.

49. For x > 0, let |x| denote the greatest integer less than or equal to x. Then

$$\lim_{x\to 0^+} x\left(\left|\frac{1}{x}\right| + \left|\frac{2}{x}\right| + \ldots + \left|\frac{10}{x}\right|\right) = \_\_\_\_\_\_\_\_\_ .$$

50. The number of subgroups of $\mathbb{Z}_7 \times \mathbb{Z}_7$ of order 7 is _______.

Q.51 – Q.60 carry two marks each.

51. Let y(x), x > 0 be the solution of the differential equation

$$x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+4y=0$$

satisfying the conditions y(1) = 1 and y'(1) = 0. Then the value of $e^2y(e)$ is _______.

52. Let T be the smallest positive real number such that the tangent to the helix

$$\cos t\ \hat{i}+\sin t\ \hat{j}+\frac{t}{\sqrt{2}}\hat{k}$$

at t = T is orthogonal to the tangent at t = 0. Then the line integral of $\vec{F}=x\hat{j}-y\hat{i}$ along the section of the helix from t = 0 to t = T is ________.

53. Let $f(x) = \frac{\sin \pi x}{\pi \sin x}$, $x \in (0, \pi)$, and let $x_0 \in (0, \pi)$ be such that $f'(x_0) = 0$. Then $(f(x_0))^2\ (1 + (\pi^2 - 1)\sin^2 x_0) =$ ________.

54. The maximum order of a permutation σ in the symmetric group S_{10} is _______.

55. Let $a_n = \sqrt{n}$, $n \geq 1$, and let $s_n = a_1 + a_2 + ... + a_n$. Then

$$\lim_{n\to\infty}\left(\frac{a_n/s_n}{-\ln(1-a_n/s_n)}\right)= \text{\_\_\_\_\_\_\_\_\_\_}.$$

56. For a real number x, define [x] to be the smallest integer greater than or equal to x. Then $\int_0^1\int_0^1\int_0^1 ([x]+[y]+[z])\,dx\,dy\,dz =$ ________.

57. For x > 1, let

$$f(x)=\int_1^x\left(\sqrt{\log t}-\frac{1}{2}\log\sqrt{t}\right)dt$$

The number of tangents to the curve y = f(x) parallel to the line x + y = 0 is ______.

58. Let α, β, γ, δ be the eigenvalues of the matrix

$$\begin{bmatrix}0&0&0&0\\1&0&0&-2\\0&1&0&1\\0&0&1&2\end{bmatrix}$$

Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 =$ __________.

59. The radius of convergence of the power series $\sum_{0}^{\infty} n!\, x^{n^2}$ is _________.

60. If $y(x) = \int_{\sqrt{x}}^{x}\frac{e^t}{t}dt,\ x>0$ then y'(1) = ________.

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
D	A	B	C	B	D	B	A	A	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	C	D	C	A	A	D	D	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	B	B	D	C	B	C	C	C	B

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
A,C	A,B,D	A,D	A,B,D	A,C	A,C	B,D	B,C	B,C,D	C,D

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	42	43	44	45	46	47	48	49	50
2	6	0	10	1260	2	0.5	3	55	8
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
3	2.09	1	30	1	3	1	6	1	1.36

Mathematics (MA)

Previous Year Solved Paper 2016

Duration : 180 minutes **Maximum Marks : 100**

Read the following instructions carefully.

1. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into **Three Sections A, B** and **C.** All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains **Numerical Answer Type Questions (NAT).** For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In Section – A (MCQ), wrong answer will result in NEGATIVE marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In Section – B (MSQ), there is NO NEGATIVE and NO PARTIAL marking provisions.There is NO NEGATIVE marking in Section – C (NAT) as well.

NOTATION

$\mathbb{N}$	The set of natural numbers = {1,2,3, …}to
$\mathbb{Z}$	The set of all integers
$\mathbb{Q}$	The set of all rational numbers
$\mathbb{R}$	The set of all real numbers
S_n	The group of permutations of n distinct symbols
$\mathbb{Z}_n$	{0,1,2,...,n – 1} with addition and multiplication modulo n
ϕ	empty set
A^T	Transpose of A
i	Imaginary number $\sqrt{-1}$
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system
∇	$\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$
I_n	Identity matrix of order n
ln	logarithm with base e

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 to Q. 10 carry one mark each.

1. The sequence $\{S_n\}$ of real numbers given by

$$s_n = \frac{\sin\frac{\pi}{2}}{1\cdot 2} + \frac{\sin\frac{\pi}{2^2}}{2\cdot 3} + \ldots + \frac{\sin\frac{\pi}{2^n}}{n\cdot(n+1)}$$

is

(A) a divergent sequence
(B) an oscillatory sequence
(C) not a Cauchy sequence
(D) a Cauchy sequence

2. Let *P* be the vector space (over $\mathbb{R}$) of all polynomials of degree ≤ 3 with real coefficients. Consider the linear transformation $T : P \to P$ defined by

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + a_2x + a_1x^2 + a_0x^3.$$

Then the matrix representation M of T with respect to the ordered basis $\{1, x, x^2, x^3\}$ satisfies

(A) $M^2 + I_4 = 0$
(B) $M^2 - I_4 = 0$
(C) $M - I_4 = 0$
(D) $M + I_4 = 0$

3. Let $f : [-1, 1] \to \mathbb{R}$ be a continuous function. Then the integral $\int_0^{\pi} x f(\sin x)\,dx$ is equivalent to

(A) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$
(B) $\frac{\pi}{2}\int_0^{\pi} f(\cos x)\,dx$
(C) $\pi\int_0^{\pi} f(\cos x)\,dx$
(D) $\pi\int_0^{\pi} f(\sin x)\,dx$

4. Let σ be an element of the permutation group S_5. Then the maximum possible order of σ is

(A) 5
(B) 6
(C) 10
(D) 15

5. Let f be a strictly monotonic continuous real valued function defined on [a,b] such that $f(a) < a$ and $f(b) > b$. Then which one of the following is TRUE ?

(A) There exists exactly one $c \in (a, b)$ such that $f(c) = c$
(B) There exists exactly two points $c_1, c_2 \in (a, b)$ such that $f(c_i) = c_i$, $i = 1,2$
(C) There exists no $c \in (a, b)$ such that $f(c) = c$
(D) There exist infinitely many points $c \in (a, b)$ such that $f(c) = c$

6. The value of $\lim\limits_{(x,y)\to(2,-2)} \frac{\sqrt{(x-y)}-2}{x-y-4}$ is

(A) 0
(B) $\frac{1}{4}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$

7. Let $\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$ and $r = |\vec{r}|$. If f(r) = ln r and g(r) = $\frac{1}{r}$, $r \neq 0$ satisfy $2\nabla f + h(r)\nabla g = \vec{0}$, then h(r) is

(A) r (B) $\frac{1}{r}$

(C) $2r$ (D) $\frac{2}{r}$

8. The nonzero value of n for which the differential equation

$$(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)\,dy = 0,\ x \neq 0,$$

becomes exact is

(A) –3 (B) –2

(C) 2 (D) 3

9. One of the points which lies on the solution curve of the differential equation

$$(y - x)\,dx + (x + y)\,dy = 0,$$

with the given condition y(0) = 1, is

(A) (1, –2) (B) (2, –1)

(C) (2, 1) (D) (–1, 2)

10. Let S be a closed subset of $\mathbb{R}$, T a compact subset of $\mathbb{R}$ such that $S \cap T \neq \phi$. Then $S \cap T$ is

(A) closed but not compact (B) not closed

(C) compact (D) neither closed nor compact

Q.11 – Q.30 carry two marks each.

11. Let S be the series $\sum_{k=1}^{\infty} \frac{1}{(2k-1)2^{(2k-1)}}$ and T be the series $\sum_{k=2}^{\infty} \left(\frac{3k-4}{3k+2}\right)^{\frac{(k+1)}{3}}$ of real numbers. Then which one of the following is TRUE?

(A) Both the series S and T are convergent

(B) S is convergent and T is divergent

(C) S is divergent and T is convergent

(D) Both the series S and T are divergent

12. Let $\{a_n\}$ be a sequence of positive real numbers satisfying

$$\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81},\quad n \geq 1,\ a_1 = 1.$$

Then all the terms of the sequence lie in

(A) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (B) [0, 1]

(C) [1, 2] (D) [1, 3]

13. The largest eigenvalue of the matrix $\begin{bmatrix} 1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4 \end{bmatrix}$ is

(A) 16
(B) 21
(C) 48
(D) 64

14. The value of the integral

$$\frac{(2n)!}{2^{2n}(n!)}\int_{-1}^{1}(1-x^2)^n\,dx, \qquad n \in \mathbb{N}$$

is

(A) $\frac{2}{(2n+1)!}$
(B) $\frac{2n}{(2n+1)!}$
(C) $\frac{2(n!)}{2n+1}$
(D) $\frac{(n+1)!}{2n+1}$

15. If the triple integral over the region bounded by the planes

$2x + y + z = 4, \quad x = 0, \quad y = 0, \quad z = 0$

is given by

$$\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz\,dy\,dx,$$

then the function $\lambda(x) - \mu(x,y)$ is

(A) $x + y$
(B) $x - y$
(C) x
(D) y

16. The surface area of the portion of the plane $y + 2z = 2$ within the cylinder $x^2 + y^2 = 3$ is

(A) $\frac{3\sqrt{5}}{2}\pi$
(B) $\frac{5\sqrt{5}}{2}\pi$
(C) $\frac{7\sqrt{5}}{2}\pi$
(D) $\frac{9\sqrt{5}}{2}\pi$

17. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x+y} & \text{if } x+y \neq 0 \\ 0 & \text{if } x+y = 0 \end{cases}$$

Then the value of $\left(\frac{\partial^2 f}{\partial x\,\partial y} + \frac{\partial^2 f}{\partial y\,\partial x}\right)$ at the point (0, 0) is

(A) 0
(B) 1
(C) 2
(D) 4

18. The function $f(x,y) = 3x^2y + 4y^3 - 3x^2 - 12y^2 + 1$ has a saddle point at

(A) (0, 0) (B) (0, 2)

(C) (1, 1) (D) (–2, 1)

19. Consider the vector field $\vec{F} = r^{\beta}(y\hat{i} - x\hat{j})$, where $\beta \in \mathbb{R}$, $\vec{r} = x\hat{i} + y\hat{j}$ and $r = |\vec{r}|$. If the absolute value of the line integral $\oint_c \vec{F}\cdot d\vec{r}$ along the closed curve C : $x^2 + y^2 = a^2$ (oriented counter clockwise) is 2π, then β is

(A) –2 (B) –1

(C) 1 (D) 2

20. Let S be the surface of the cone $z = \sqrt{x^2+y^2}$ bounded by the planes z = 0 and z = 3. Further, let C be the closed curve forming the boundary of the surface S. A vector field $\vec{F}$ is such that $\nabla \times \vec{F} = x\hat{i} - y\hat{j}$. The absolute value of the line integral $\oint_c \vec{F}\cdot d\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, is

(A) 0 (B) 9π

(C) 15π (D) 18π

21. Let y(x) be the solution of the differential equation

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) = x;\ y(1) = 0,\ \left.\frac{dy}{dx}\right|_{x=1} = 0.$$

Then y(2) is

(A) $\frac{3}{4} + \frac{1}{2}\ln 2$ (B) $\frac{3}{4} - \frac{1}{2}\ln 2$

(C) $\frac{3}{4} + \ln 2$ (D) $\frac{3}{4} - \ln 2$

22. The general solution of the differential equation with constant coefficients

$$\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

approaches zero as $x \to \infty$, if

(A) b is negative and c is positive (B) b is positive and c is negative

(C) both b and c are positive (D) both b and c are negative

23. Let $S \subset \mathbb{R}$ and ∂S denote the set of points x in $\mathbb{R}$ such that every neighbourhood of x contains some points of S as well as some points of complement of S. Further, let $\overline{S}$ denote the closure of S. Then which one of the following is FALSE?

(A) $\partial\mathbb{Q} = \mathbb{R}$

(B) $\partial(\mathbb{R}\setminus T) = \partial T,\ T \subset \mathbb{R}$

(C) $\partial(T \cup V) = \partial T, \cup \partial V,\ T, V \subset \mathbb{R},\ T \cap V \neq \phi$

(D) $\partial T = \overline{T} \cap (\overline{\mathbb{R} \setminus T}),\ T \subset \mathbb{R}$

24. The sum of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+n-2}$$

is

(A) $\frac{1}{3}\ln 2 - \frac{5}{18}$ (B) $\frac{1}{3}\ln 2 - \frac{5}{6}$

(C) $\frac{2}{3}\ln 2 - \frac{5}{18}$ (D) $\frac{2}{3}\ln 2 - \frac{5}{6}$

25. Let $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-1|}$ for all $x \in [-1, 1]$. Then which one of the following is TRUE?

(A) Maximum value of f(x) is $\frac{3}{2}$ (B) Minimum value of f(x) is $\frac{1}{3}$

(C) Maximum of f(x) occurs at $x = \frac{1}{2}$ (D) Minimum of f(x) occurs at x = 1

26. The matrix $M = \begin{bmatrix} \cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha \end{bmatrix}$ is a unitary matrix when α is

(A) $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ (B) $(3n+1)\frac{\pi}{3}, n \in \mathbb{Z}$

(C) $(4n+1)\frac{\pi}{4}, n \in \mathbb{Z}$ (D) $(5n+1)\frac{\pi}{5}, n \in \mathbb{Z}$

27. Let $M = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0 \end{bmatrix}$, $\alpha \in \mathbb{R} \setminus \{0\}$ and **b** a non-zero vector such that $Mx = \mathbf{b}$ for some $x \in \mathbb{R}^3$.

Then the value of $\mathbf{x}^T \mathbf{b}$ is

(A) $-\alpha$ (B) α

(C) 0 (D) 1

28. The number of group homomorphisms from the cyclic group $\mathbb{Z}_4$ to the cyclic group $\mathbb{Z}_7$ is

(A) 7 (B) 3

(C) 2 (D) 1

29. In the permutation group S_n ($n \geq 5$), if H is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?

(A) Order of H is 2 (B) Index of H in S_n is 2

(C) H is abelian (D) $H = S_n$

30. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x(1 + x^{\alpha} \sin(\ln x^2)) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then, at x = 0, the function f is

(A) continuous and differentiable when $\alpha = 0$

(B) continuous and differentiable when $\alpha > 0$

(C) continuous and differentiable when $-1 < \alpha < 0$

(D) continuous and differentiable when $\alpha < -1$

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

31. Let $\{s_n\}$ be a sequence of positive real numbers satisfying

$$2s_{n+1} = s_n^2 + \frac{3}{4}, \quad n \geq 1.$$

If α and β are the roots of the equation $x^2 - 2x + \frac{3}{4} = 0$ and $\alpha < s_1 < \beta$, then which of the following statement(s) is (are) TRUE?

(A) $\{s_n\}$ is monotonically decreasing

(B) $\{s_n\}$ is monotonically increasing

(C) $\lim_{n\to\infty} s_n = \alpha$

(D) $\lim_{n\to\infty} s_n = \beta$

32. The value(s) of the integral

$$\int_{-\pi}^{\pi} |x| \cos nx \, dx, \quad n \geq 1$$

is (are)

(A) 0 when n is even

(B) 0 when n is odd

(C) $-\frac{4}{n^2}$ when n is even

(D) $-\frac{4}{n^2}$ when n is odd

33. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{xy}{|x|} & \text{if } x \neq 0 \\ 0 & \text{elsewhere} \end{cases}$$

Then at the point (0,0), which of the following statement(s) is (are) TRUE ?

(A) f is not continuous

(B) f is continuous

(C) f is differentiable

(D) Both first order partial derivatives of f exist

34. Consider the vector field $\vec{F} = x\hat{i} + y\hat{j}$ on an open connected set $S \subset \mathbb{R}^2$. Then which of the following statement(s) is (are) TRUE ?

(A) Divergence of $\vec{F}$ is zero on S

(B) The line integral of $\vec{F}$ is independent of path in S

(C) $\vec{F}$ can be expressed as a gradient of a scalar function on S

(D) The line integral of $\vec{F}$ is zero around any piecewise smooth closed path in S

35. Consider the differential equation

$$\sin 2x \frac{dy}{dx} = 2y + 2\cos x, y\left(\frac{\pi}{4}\right) = 1 - \sqrt{2}.$$

Then which of the following statement(s) is(are) TRUE?

(A) The solution is unbounded when $x \to 0$

(B) The solution is unbounded when $x \to \frac{\pi}{2}$

(C) The solution is bounded when $x \to 0$

(D) The solution is bounded when $x \to \frac{\pi}{2}$

36. Which of the following statement(s) is(are) TRUE?

(A) There exists a connected set in $\mathbb{R}$ which is not compact

(B) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact

(C) Arbitrary union of closed intervals in $\mathbb{R}$ is always closed

(D) Every bounded infinite subset V of $\mathbb{R}$ has a limit point in V itself

37. Let $P(x) = \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x - 1$ for all $x \in \mathbb{R}$. Then which of the following statement(s) is(are) TRUE?

(A) The equation P(x) = 0 has exactly one solution in $\mathbb{R}$

(B) P(x) is strictly increasing for all $x \in \mathbb{R}$

(C) The equation P(x) = 0 has exactly two solutions in $\mathbb{R}$

(D) P(x) is strictly decreasing for all $x \in \mathbb{R}$

38. Let G be a finite group and o(G) denotes its order. Then which of the following statement(s) is(are) TRUE?

(A) G is abelian if o(G) = pq where p and q are distinct primes

(B) G is abelian if every non identity element of G is of order 2

(C) G is abelian if the quotient group $\frac{G}{Z(G)}$ is cyclic, where Z(G) is the center of G

(D) G is abelian if $o(G) = p^3$, where p is prime

39. Consider the set V = $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \middle| \ \alpha x + \beta y + z = \gamma,\ \alpha, \beta, \gamma \in \mathbb{R} \right\}$. For which of the following choice(s) the set V becomes a two dimensional subspace of $\mathbb{R}^3$ over $\mathbb{R}$?

(A) $\alpha = 0,\ \beta = 1,\ \gamma = 0$ (B) $\alpha = 0,\ \beta = 1,\ \gamma = 1$

(C) $\alpha = 1,\ \beta = 0,\ \gamma = 0$ (D) $\alpha = 1,\ \beta = 1,\ \gamma = 0$

40. Let S = $\left\{ \frac{1}{3^n} + \frac{1}{7^m} \middle| \ n, m \in \mathbb{N} \right\}$. Then which of the following statement(s) is(are) TRUE?

(A) S is closed (B) S is not open

(C) S is connected (D) 0 is a limit point of S

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

Q.41 – Q.50 carry one mark each.

41. Let $\{S_n\}$ be a sequence of real numbers given by

$$s_n = 2^{(-1)^n}\left(1 - \frac{1}{n}\right)\sin\frac{n\pi}{2}, \quad n \in \mathbb{N}.$$

Then the least upper bound of the sequence $\{S_n\}$ is __________.

42. Let $\{S_k\}$ be a sequence of real numbers, where $s_k = k^{\alpha/k}$, $k \geq 1$, $\alpha > 0$. Then $\lim\limits_{n \to \infty}(s_1 s_2 \ldots s_n)^{1/n}$ is __________.

43. Let x = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$ be a non-zero vector and A = $\frac{xx^T}{x^Tx}$. Then the dimension of the vector space $\{y \in \mathbb{R}^3 \mid Ay = 0\}$ over $\mathbb{R}$ is ________.

44. Let f be a real valued function defined by

$f(x,y) = 2\ln(x^2y^2e^{y/x})$, $x > 0$, $y > 0$.

Then the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ at any point (x,y), where $x > 0$, $y > 0$, is _________.

45. Let $\vec{F} = \sqrt{x}\,\hat{i} + (x + y^3)\hat{j}$ be a vector field for all (x,y) with $x \geq 0$ and $\vec{r} = x\hat{i} + y\hat{j}$. Then the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$ from (0,0) to (1,1) along the path C : $x = t^2$, $y = t^3$, $0 \leq t \leq 1$ is _______.

46. If $f : (-1, \infty) \to \mathbb{R}$ defined by $f(x) = \dfrac{x}{1+x}$ is expressed as

$$f(x) = \frac{2}{3} + \frac{1}{9}(x-2) + \frac{c(x-2)^2}{(1+\xi)^3},$$

where ξ lies between 2 and x, then the value of c is ___________.

47. Let $y_1(x)$, $y_2(x)$ and $y_3(x)$ be linearly independent solutions of the differential equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0.$$

If the Wronskian $W(y_1, y_2, y_3)$ is of the form ke^{bx} for some constant k, then the value of b is ________.

48. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-4)^n}{n(n+1)}(x+2)^{2n}$ is ________.

49. Let $f : (0, \infty) \to \mathbb{R}$ be a continuous function such that

$$\int_0^x f(t)\,dt = -2 + \frac{x^2}{2} + 4x\sin 2x + 2\cos 2x.$$

Then the value of $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$ is ________.

50. Let G be a cyclic group of order 12. Then the number of non-isomorphic subgroups of G is ______.

Q.51 – Q.60 carry two marks each.

51. The value of $\lim_{n\to\infty}\left(8n - \frac{1}{n}\right)^{\frac{(-1)^n}{n^2}}$ is equal to ________.

52. Let R be the region enclosed by $x^2 + 4y^2 \geq 1$ and $x^2 + y^2 \leq 1$. Then the value of $\iint_R |xy|\,dx\,dy$ is ________.

53. Let

$$M = \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{bmatrix}, \alpha\beta\gamma = 1,\ \alpha, \beta, \gamma \in \mathbb{R} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3.$$

Then Mx = 0 has infinitely many solutions if trace (M) is ________.

54. Let C be the boundary of the region enclosed by $y = x^2$, $y = x + 2$, and $x = 0$. Then the value of the line integral

$$\oint_C (xy - y^2)dx - x^3dy,$$

where C is traversed in the counter clockwise direction, is _______.

55. Let S be the closed surface forming the boundary of the region V bounded by $x^2 + y^2 = 3$, $z = 0$, $z = 6$. A vector field $\vec{F}$ is defined over V with $\nabla . \vec{F} = 2y + z + 1$. Then the value of

$$\frac{1}{\pi}\iint_S \vec{F} \cdot \hat{n} \, ds,$$

where $\hat{n}$ is the unit outward drawn normal to the surface S, is ________.

56. Let y(x) be the solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0, \quad y(0) = 1, \quad \left.\frac{dy}{dx}\right|_{x=0} = -1.$$

Then y(x) attains its maximum value at x = _______.

57. The value of the double integral $\int_0^{\pi}\int_0^{x} \frac{\sin y}{\pi - y} dy\, dx$ is _______.

58. Let H denote the group of all 2 × 2 invertible matrices over $\mathbb{Z}_5$ under usual matrix multiplication.

Then the order of the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ in H is ______.

59. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix}$, N(A) the null space of A and R(B) the range space of B. Then the dimension of $N(A) \cap R(B)$ over $\mathbb{R}$ is _______.

60. The maximum value of $f(x,y) = x^2 + 2y^2$ subject to the constraint $y - x^2 + 1 = 0$ is _____.

ANSWER KEY

SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
D	B	A	B	A	B	C	D	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	D	B	C	D	A	B	D	A	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	C	C	C	A	A	C	D	B	B

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

31	32	33	34	35	36	37	38	39	40
A,C	A,D	B,D	B,C,D	C,D	A,B	A,D	B,C	A,C,D	B,D

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

41	42	43	44	45	46	47	48	49	50
0.5	1	2	8	1.5	-1	6	0.5	0.25	6
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
1	0.375	3	0.8	72	-0.28	2	3	1	2

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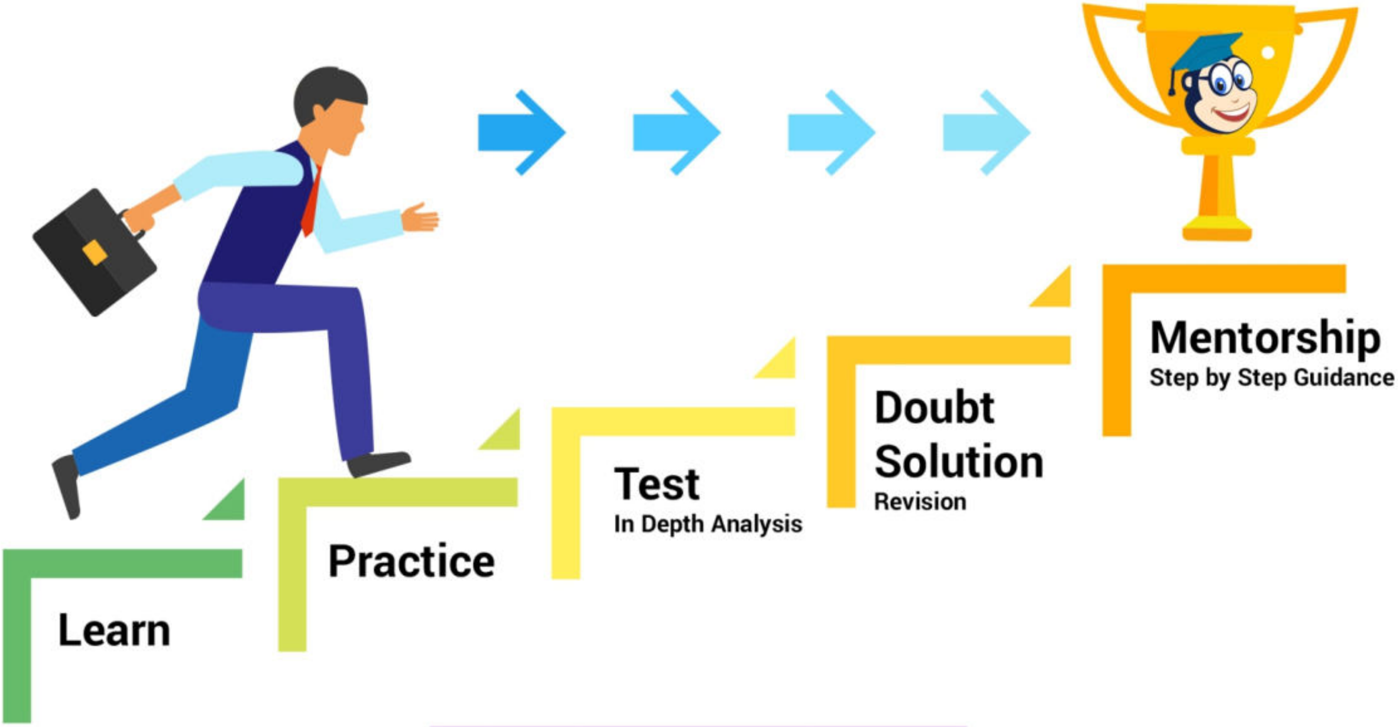

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