Series will converge if integration exists (to a finite value)

(1) $\int_0^2 \frac{1}{2x-x^2}\,dx$

$= \int_2^0 \frac{1}{x^2-2x}\,dx$

$= \int_2^0 \frac{1}{x^2-2x+1-1}\,dx \Rightarrow \int_2^0 \frac{1}{(x-1)^2-1}\,dx$

$\Rightarrow \frac{1}{2}\left[\ln\left(\frac{x-1-1}{x-1+1}\right)\right]_2^0 + C$

$\Rightarrow \frac{1}{2}\left[\frac{\ln(x-2)}{\ln x}\right]_2^0 + C$

When we put limit 0 $\Rightarrow$ then numerator will be $\ln(0-2)$ and log of negative no does not exist (defined) so series is not convergent.

(2) $\int_0^1 \frac{1}{\sqrt{1-x^3}}\,dx$

Let $1-x^3 = t^2$ $\qquad x=0, t=1$

$-3x^2\,dx = 2t\,dt$ $\qquad x=1, t=0$

$3(1-t^2)^{2/3} = 2t\,dt$

$\Rightarrow dx \Rightarrow \int \frac{2t\,dt}{-3(1-t^2)^{2/3}}$

let $1-t^2 = k$ $\qquad t=1 \quad k=0$

$-2t\,dt = dk$ $\qquad t=0 \quad k=1$

$\Rightarrow \int_0^1 \frac{dk}{3k^{2/3}}$

$\Rightarrow$ Integration will give a finite value so Convergent.

(2) $\int_1^\infty \frac{1}{x\sqrt{x^2+1}}\,dx$

$x^2+1 = t^2$ $\qquad x=1 \quad t=\sqrt{2}$

$2x\,dx = 2t\,dt$ $\qquad x=\infty \quad t=\infty$

$dx = \frac{t\,dt}{x}$

$\int_{\sqrt{2}}^\infty \frac{t\,dt}{x \cdot x \cdot t}$

$\int_{\sqrt{2}}^\infty \frac{dt}{t^2-1}$

$\Rightarrow \left[\frac{\ln(t-1)}{\ln(t+1)}\right]_{\sqrt{2}}^\infty + C$

$\ln \infty = \infty$ so divergent.