(1) $1 + \frac{x}{1} + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{x^7}{7} + \cdots$

from 3rd term →

$$a_n = \frac{1\cdot 3\cdot 5 \cdots (2n-1)}{2\cdot 4\cdot 6 \cdots (2n)}\cdot\frac{x^{2n+1}}{2n+1}, \quad n = 1, 2, 3, \cdots$$

[1, 3, 5 ⟶ A.P.
n^{th} term $= 1 + (n-1)2$
$= 2n-1$

2, 4, 6 ⟶ A.P
n^{th} term $= 2n$]

Check → $n = 1 \rightarrow a_1 = \frac{1}{2}\,x^3/3$

$a_2 = \frac{1\cdot 3}{2\cdot 4}\,x^5/5$

& so on.

Thursday

$$x + \frac{1}{2}\cdot\frac{x^2}{4} + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{x^4}{8} + \frac{1\cdot3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8\cdot10}\cdot\frac{x^6}{12} + \cdots$$

from 2nd term → exponent of x is

$2, 4, 6, \ldots$

∴ in n^{th} term exponent will be $2n$

denominator of x is twice the exponent of x.

∴ n^{th} term of (x - expression) $= \frac{x^{2n}}{4n}$

Other term is like

$\frac{1}{2}$ · $\frac{1\cdot3\cdot5}{2\cdot4\cdot6}$ · $\frac{1\cdot3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8\cdot10}$ ⋯

(two terms extra every time)

no of terms → 1, 3, 5

∴ no. of terms in n^{th} expression $= 1 + (n-1)2$

$= 2n - 1$

Tuesday

$(2n-1)^{th}$ term of $1, 3, 5, \ldots$

$= 1 + (2n-1-1)2 = 4n-3$

Similarly, $(2n-1)^{th}$ term of $2, 4, 6, \ldots$

$= 2 + (2n-1-1)2$

$= 4n-2$

$$\therefore a_n = \frac{1.3.5. \ldots (4n-3)}{2.4.6 \ldots (4n-2)} \frac{x^{2n}}{4n}$$

Put $n=1$: $a_1 = \frac{1}{2} \frac{x^2}{4}$

$4n-3 = 1$, $4n-2 = 2$, $\therefore n = 1$

$$a_2 = \frac{1.3.5}{2.4.6} \frac{x^4}{8}$$

$4n-3 = 5$, $4n-2 = 6$, $\therefore n = 2$