MATHS

The value of a for which $x^3 - 3x + a = 0$ has two distinct roots in $[0, 1]$ is given by

A -1

B 1

C 3

D does not exists

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ANSWER

Given $x^3 - 3x + a = 0$

Let $f(x) = x^3 - 3x + a$

$\implies f'(x) = 3x^2 - 3 = 0$

$\implies x^2 - 1 = 0$

$\implies x = \pm 1$ are the critical points.

$f''(x) = 6x$

On substituting the critical points in $f''(x)$ we get

$f''(1) = 6 > 0 \implies x = 1$ is the local minimum and

$f''(-1) = -6 < 0 \implies x = -1$ is the local maximum

For $f(x)$ to have two distinct roots we must have either $f(1) = 0$ or $f(-1) = 0$. Since, to have real distinct roots, the function should touch the X-axis just once and intersect it once.

$f(1) = 1 - 3 + a = 0 \implies a = 2$ and

$f(-1) = -1 + 3 + a = 0 \implies a = -2$

Therefore, to have distinct roots, the value of a should be in $[-2, 2]$

Let $a = 0 \in [-2, 2]$

Therefore, $f(x) = x^3 - 3x + a$

$\implies x^3 - 3x + 0 = 0$

$$\implies x^3 - 3x + 0 = 0$$

$$\implies x(x^2 - 3) = 0$$

$$\implies x = 0 \in [0, 1] \text{ or } x = \sqrt{3} \notin [0, 1] \text{ or } x = -\sqrt{3} \notin [0, 1]$$

Therefore, only one root is lying in $[0, 1]$. Hence, a value doesn't exist to have two roots in the interval $[0, 1]$