

use method of change of variable

$$\text{let } xy = a \quad \& \quad x^2 + y^2 = b$$

$$\text{then } 1 \leq xy \leq 4 \quad \& \quad -3 \leq x^2 + y^2 \leq 3$$

$$\boxed{1 \leq a \leq 4 \quad \& \quad -3 \leq b \leq 3}$$

$$\frac{1}{|J|} = \begin{vmatrix} \frac{\partial a}{\partial x} & \frac{\partial a}{\partial y} \\ \frac{\partial b}{\partial x} & \frac{\partial b}{\partial y} \end{vmatrix} = \begin{vmatrix} y & x \\ 2x & -2y \end{vmatrix} = -2y^2 - 2x^2$$

$$\boxed{|J| = \frac{1}{2(x^2 + y^2)}} \rightarrow \text{jacobian}$$

Now

$$\int \int xy (x^2 + y^2) dx dy = \int \int a (x^2 + y^2) |J| da db$$

$$= \int_{b=-3}^3 \int_{a=1}^4 a \cancel{(x^2 + y^2)} \times \frac{1}{2 \cancel{(x^2 + y^2)}} da db$$

$$\Rightarrow \frac{(3+3)}{2 \times 2} (a^2) \Big|_1^4 \Rightarrow \frac{3 \times 6 \times 15}{4 \times 2} \Rightarrow \boxed{\frac{45}{2}} \text{ h}$$